CRITICAL THINKING

Lecture #17

Advanced Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

1. $p \rightarrow q.$
2. $p.$

$\therefore\ q.$

2. *Modus Tollens* (M.T.)

1. $p \rightarrow q.$
2. $\sim q.$

$\therefore\ \sim p.$

3. Hypothetical Syllogism (H.S.)

1. $p \rightarrow q.$
2. $q \rightarrow r.$

$\therefore\ p \rightarrow r.$

4. Disjunctive Syllogism (D.S.)

1. $p \vee q.$
2. $\sim p.$

$\therefore\ q.$

5. Constructive Dilemma (C.D.)

1. $(p \rightarrow q)\ \&\ (r \rightarrow s).$
2. $p \vee r.$

$\therefore\ q \vee s.$

6. Absorption (Abs.)

1. $p \rightarrow q.$

$\therefore\ p \rightarrow (p\ \&\ q).$

7. Simplification (Simp.)

1. $p\ \&\ q.$

$\therefore\ p.$

8. Conjunction (Conj.)

1. $p.$
2. $q.$

$\therefore\ p\ \&\ q.$

9. Addition (Add.)

1. $p.$

$\therefore\ p \vee q.$

Natural Deduction: *Instructions*

Proving the validity of an argument using natural deduction works as follows:

1. Translate the argument (if it is in English) into the language of symbolic logic,
2. Put the argument into argumentative form, and
3. Use the nine rules of inference to derive the conclusion from the premises.

Natural Deduction

Today we finally bring all of our skills in natural deduction together. We now look at proofs where we do not know in advance how many steps they will take to solve. However, the process remains the same.

Example

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. A → B.
2. A v (C & D).
3. ~B & ~E.

∴ C.

Example (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. A → B.
2. A v (C & D).
3. ~B & ~E.

∴ C.

4.	~B.	3; Simp.
5.	~A.	1, 4; M.T.
6.	C & D.	2, 5; D.S.
7.	C.	6; Simp.

Argument #1

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. $(\sim M \,\&\, \sim N) \rightarrow (O \rightarrow N)$.
2. $N \rightarrow M$.
3. $\sim M$.

$\therefore$ $\sim O$.

Argument #1 (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. (~M & ~N) → (O → N).
2. N → M.
3. ~M.

∴ ~O.

4.	~N.	2, 3; M.T.
5.	~M & ~N.	3, 4; Conj.
6.	O → N.	1, 5; M.P.
7.	~O.	6, 4; M.T.

Natural Deduction

As always, the real goal with natural deduction is to be able to take arguments in English, translate them into the language of logic, and then formally prove their validity.

Argument #2

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct this argument's formal proof of validity.

Layli is present only if **Majnun** is happy. **Cala** is pleased if both **Layli** is present and **Majnun** is happy. **Dirran** being pleased is a necessary condition for both **Layli** being present and **Cala** being pleased. Therefore, **Layli** being present is sufficient for **Dirran** to be pleased. (L, M, C, D)

Argument #2 (Solution)

Layli is present only if **Majnun** is happy. **Cala** is pleased if both **Layli** is present and **Majnun** is happy. **Dirran** being pleased is a necessary condition for both **Layli** being present and **Cala** being pleased. Therefore, **Layli** being present is sufficient for **Dirran** to be pleased. (L, M, C, D)

1.	$L \rightarrow M$.	
2.	$(L \,\&\, M) \rightarrow C$.	
3.	$(L \,\&\, C) \rightarrow D$.	
∴	$L \rightarrow D$.	
4.	$L \rightarrow (L \,\&\, M)$.	1; Abs.
5.	$L \rightarrow C$.	4, 2; H.S.
6.	$L \rightarrow (L \,\&\, C)$.	5; Abs.
7.	$L \rightarrow D$.	6, 3; H.S.

Next Class…

We will have an in-class review session for **unit exam #2**.