CRITICAL THINKING

Lecture #16

Longer Proofs by Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

1. $p \rightarrow q.$
2. $p.$

$\therefore\ q.$

2. *Modus Tollens* (M.T.)

1. $p \rightarrow q.$
2. $\sim q.$

$\therefore\ \sim p.$

3. Hypothetical Syllogism (H.S.)

1. $p \rightarrow q.$
2. $q \rightarrow r.$

$\therefore\ p \rightarrow r.$

4. Disjunctive Syllogism (D.S.)

1. $p \vee q.$
2. $\sim p.$

$\therefore\ q.$

5. Constructive Dilemma (C.D.)

1. $(p \rightarrow q)\ \&\ (r \rightarrow s).$
2. $p \vee r.$

$\therefore\ q \vee s.$

6. Absorption (Abs.)

1. $p \rightarrow q.$

$\therefore\ p \rightarrow (p\ \&\ q).$

7. Simplification (Simp.)

1. $p\ \&\ q.$

$\therefore\ p.$

8. Conjunction (Conj.)

1. $p.$
2. $q.$

$\therefore\ p\ \&\ q.$

9. Addition (Add.)

1. $p.$

$\therefore\ p \vee q.$

Natural Deduction: *Instructions*

Proving the validity of an argument using natural deduction works as follows:

1. Translate the argument (if it is in English) into the language of symbolic logic,
2. Put the argument into argumentative form, and
3. Use the nine rules of inference to derive the conclusion from the premises.

Natural Deduction

We have been constructing proofs where the steps are not given to us. To continue this process, today we now look at arguments whose proofs can be done in just *three* steps. Once you can do this, you will have the skills to tackle most proofs by natural deduction.

Example

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $T \rightarrow U$.
2. $V \vee \sim U$.
3. $\sim V \ \& \sim W$.

$\therefore \sim T$.

Example (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $T \rightarrow U$.
2. $V \vee \sim U$.
3. $\sim V \;\&\; \sim W$.

∴ $\sim T$.

4.	$\sim V$.	3; Simp.
5.	$\sim U$.	2, 4; D.S.
6.	$\sim T$.	1, 5; M.T.

Argument #1

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(A \vee B) \rightarrow \sim C$.
2. $C \vee D$.
3. A.

$\therefore D$.

Argument #1 (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(A \vee B) \rightarrow {\sim}C$.
2. $C \vee D$.
3. A.

$\therefore$ D.

4.	$A \vee B$.	3; Add.
5.	${\sim}C$.	1, 4; M.P.
6.	D.	2, 5; D.S.

Argument #2

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(P \rightarrow Q) \,\&\, (Q \rightarrow P)$.
2. $R \rightarrow S$.
3. $P \vee R$.

$\therefore$ $Q \vee S$.

Argument #2 (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(P \rightarrow Q) \ \& \ (Q \rightarrow P)$.
2. $R \rightarrow S$.
3. $P \vee R$.

$\therefore$ $Q \vee S$.

4.	$P \rightarrow Q$.	1; Simp.
5.	$(P \rightarrow Q) \ \& \ (R \rightarrow S)$.	4, 2; Conj.
6.	$Q \vee S$.	5, 3; C.D.

Natural Deduction

As I have said before, the real goal with natural deduction is to be able to take arguments in English, translate them into the language of logic, and then formally prove their validity.

Argument #3

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

Majnun being happy is necessary for **Layli** being present. **Majnun** being happy is sufficient for either **Cala** or **Dirran** being pleased. **Cala** is not pleased. **Layli** is present. Therefore, **Dirran** is pleased. (L, M, C, D)

Argument #3 (Solution)

Majnun being happy is necessary for **Layli** being present. **Majnun** being happy is sufficient for either **Cala** or **Dirran** being pleased. **Cala** is not pleased. **Layli** is present. Therefore, **Dirran** is pleased. (L, M, C, D)

1. $L \rightarrow M$.
2. $M \rightarrow (C \vee D)$.
3. $\sim C$.
4. L.

∴ D.

Either proof is perfectly acceptable.

5.	M.	1, 4; M.P.
6.	$C \vee D$.	2, 5; M.P.
7.	D.	6, 3; D.S.

5.	$L \rightarrow (C \vee D)$.	1, 2; H.S.
6.	$C \vee D$.	5, 4; M.P.
7.	D.	6, 3; D.S.

Next Class…

We will do a workshop on creating formal proofs of validity that can be done in either two or three steps.