### **CRITICAL THINKING** Lecture #16

Longer Proofs by Natural Deduction

#### Professor David Emmanuel Gray







#### The Nine Rules of Inference

I. Modus Ponens (M.P.)

2. *Modus Tollens* (M.T.)

I. 
$$p \rightarrow q$$
.
 I.  $p \rightarrow q$ .

 2.  $p$ .
  $2. \sim q$ .

  $\therefore q$ .
  $2. \sim q$ .

  $\therefore -p$ .

#### 4. Disjunctive Syllogism (D.S.)

I.  $(p \rightarrow q) \& (r \rightarrow s)$ . I.  $p \lor q$ . 2.  $p \lor r$ . 2. ~p. :. q.  $\therefore q \lor s.$ 

#### 7. Simplification (Simp.)

$$\frac{1. p \& q.}{\therefore p.}$$

8. Conjunction (Conj.)

I. 
$$p$$
.  
2.  $q$ .  
 $\therefore p \& q$ .

#### 3. Hypothetical Syllogism (H.S.)

I. 
$$p \rightarrow q$$
.  
2.  $q \rightarrow r$ .  
 $\therefore p \rightarrow r$ .

5. Constructive Dilemma (C.D.)

$$\begin{array}{ccc} \mathbf{I}. & p \rightarrow q. \\ \hline \ddots & p \rightarrow (p \& q). \end{array} \end{array}$$

9. Addition (Add.)

$$\frac{\text{I.} \quad p.}{\therefore \quad p \lor q.}$$

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### Natural Deduction: Instructions

Proving the validity of an argument using natural deduction works as follows:

- Translate the argument (if it is in English) into the language of symbolic logic, I.
- Put the argument into argumentative form, and 2.
- Use the nine rules of inference to derive the conclusion from the premises. 3.



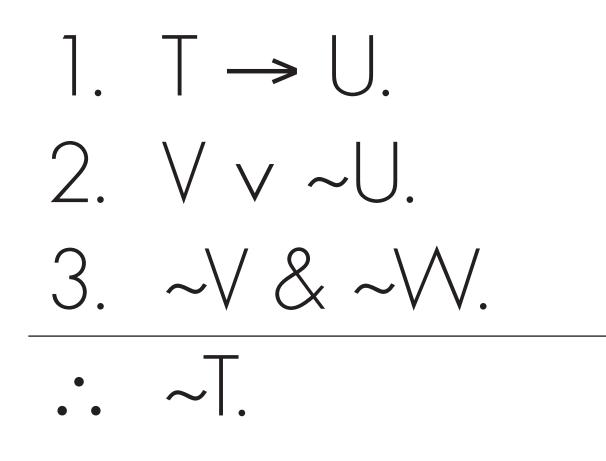
### Natural Deduction

We have been constructing proofs where the steps are not given to us. To continue this process, today we now look at arguments whose proofs can be done in just *three* steps. Once you can do this, you will have the skills to tackle most proofs by natural deduction.



### Example

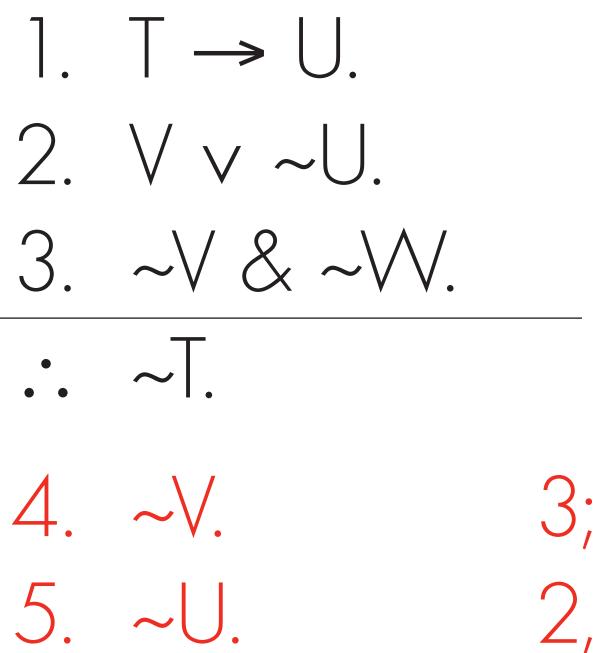
proof of validity. This proof can be done in only *three* steps.





### Example (Solution)

proof of validity. This proof can be done in only *three* steps.



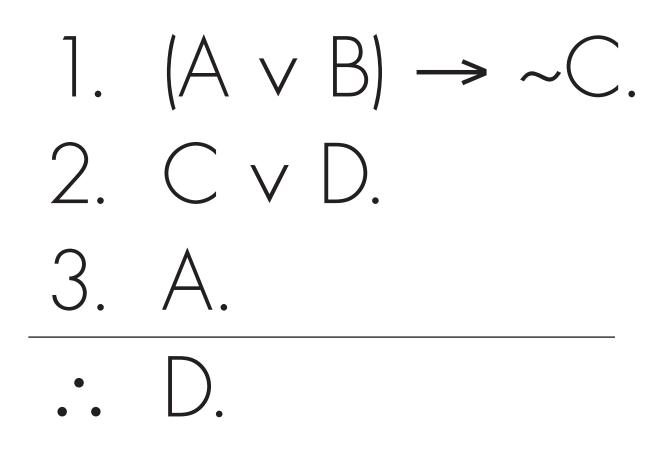
6. ~T.

3; Simp. 2, 4; D.S. 1, 5; M.T.



Argument #1

proof of validity. This proof can be done in only *three* steps.



### Argument #1 (Solution)

proof of validity. This proof can be done in only *three* steps.

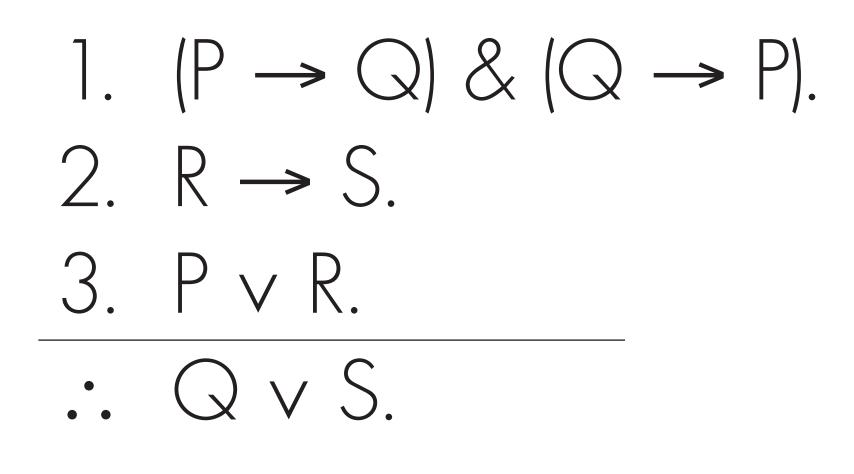
- 1.  $(A \lor B) \rightarrow \sim C$ . 2. C v D.
- 3. A. . D.
- 4. A v B. 5. ~C. 6. D.

3; Add. 1, 4; M.P. 2, 5; D.S.



Argument #2

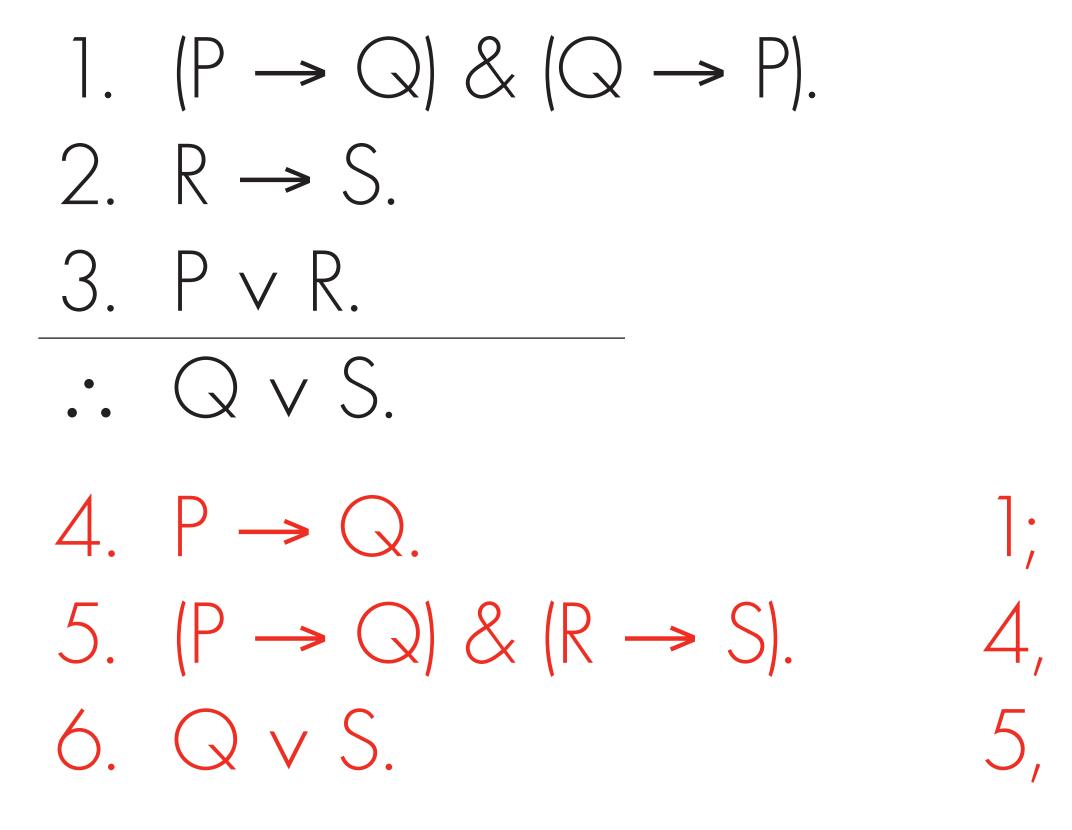
proof of validity. This proof can be done in only *three* steps.





Argument #2 (Solution)

proof of validity. This proof can be done in only *three* steps.



# The following is a valid argument. Use natural deduction to construct this argument's formal

I; Simp. 4, 2; Conj. 5, 3; C.D.



### Natural Deduction

As I have said before, the real goal with natural deduction is to be able to take arguments in English, translate them into the language of logic, and then formally prove their validity.

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Argument #3

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *three* steps.

Majnun being happy is necessary for Layli being present. Majnun being happy is sufficient for either Cala or Dirran being pleased. Cala is not pleased. Layli is present. Therefore, Dirran is pleased. (L, M, C, D)

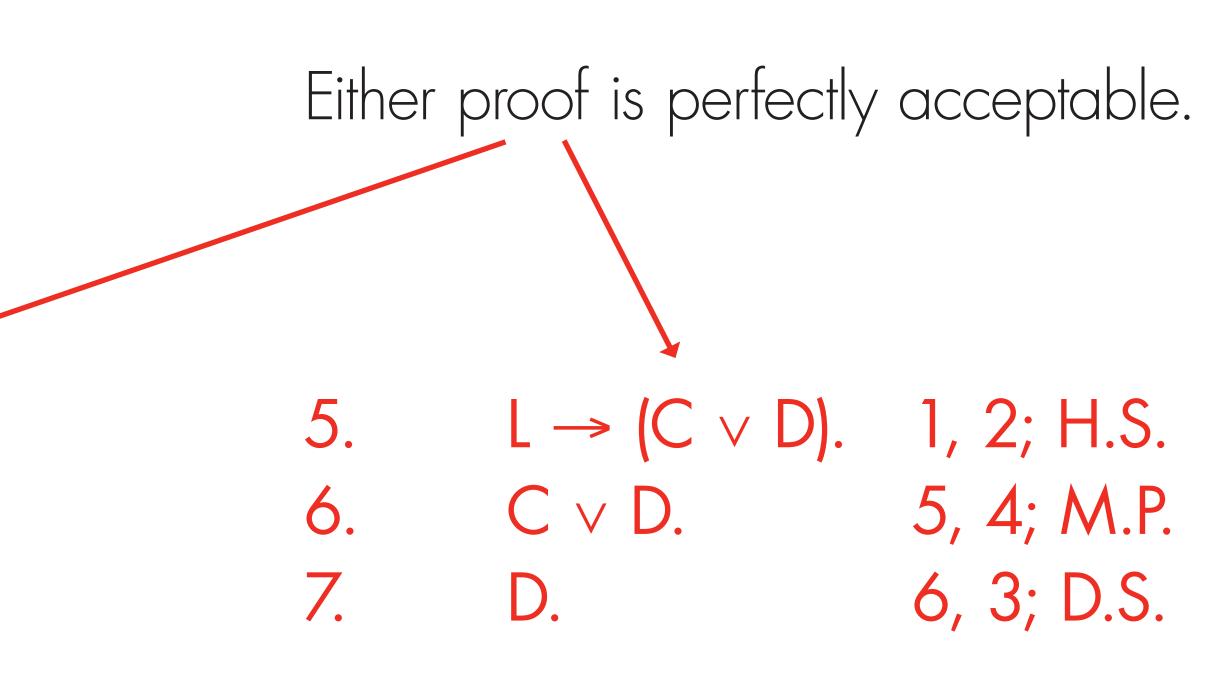




### Argument #3 (Solution)

Majnun being happy is necessary for Layli being present. Majnun being happy is sufficient for either Cala or Dirran being pleased. Cala is not pleased. Layli is present. Therefore, Dirran is pleased. (L, M, C, D)

1.	$L \rightarrow M.$	
2.	$M \rightarrow (C \lor D)$	
3.	~C.	
4.	L.	
•	D.	
5.	M.	1, 4; M.P.
6.	$C \lor D.$	2, 5; M.P.
7.	D.	6, 3; D.S.



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### We will do a workshop on creating formal proofs of validity that can be done in either two or three steps.

