CRITICAL THINKING

Lecture #15

Creating Proofs by Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

1. $p \rightarrow q$.
2. p.

$\therefore$ q.

2. *Modus Tollens* (M.T.)

1. $p \rightarrow q$.
2. $\sim q$.

$\therefore$ $\sim p$.

3. Hypothetical Syllogism (H.S.)

1. $p \rightarrow q$.
2. $q \rightarrow r$.

$\therefore$ $p \rightarrow r$.

4. Disjunctive Syllogism (D.S.)

1. $p \vee q$.
2. $\sim p$.

$\therefore$ q.

5. Constructive Dilemma (C.D.)

1. $(p \rightarrow q) \,\&\, (r \rightarrow s)$.
2. $p \vee r$.

$\therefore$ $q \vee s$.

6. Absorption (Abs.)

1. $p \rightarrow q$.

$\therefore$ $p \rightarrow (p \,\&\, q)$.

7. Simplification (Simp.)

1. $p \,\&\, q$.

$\therefore$ p.

8. Conjunction (Conj.)

1. p.
2. q.

$\therefore$ $p \,\&\, q$.

9. Addition (Add.)

1. p.

$\therefore$ $p \vee q$.

Natural Deduction: *Instructions*

Proving the validity of an argument using natural deduction works as follows:

1. Translate the argument (if it is in English) into the language of symbolic logic,
2. Put the argument into argumentative form, and
3. Use the nine rules of inference to derive the conclusion from the premises.

Natural Deduction

So far we have just seen formal proofs that only require being able to recognize the rules of inference (i.e., the "patterns") being applied. Now we can begin to begin to construct longer proofs where the steps are not given to us. To introduce you to this process, today we will look at arguments whose proofs can be done in just *two* steps. While these proofs may be short, they will develop the skills necessary for longer and more complicated proofs.

Argument #1

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *two* steps.

1. A.
2. B.

∴ (A ∨ C) & B.

Argument #1 (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *two* steps.

1. A.
2. B.

∴ (A v C) & B.

3. A v C. 1; Add.
4. (A v C) & B. 3, 2; Conj.

Argument #2

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *two* steps.

1. $A \rightarrow B$.
2. $A \vee C$.
3. $C \rightarrow D$.

$\therefore$ $B \vee D$.

Argument #2 (Solution)

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *two* steps.

1. $A \rightarrow B$.
2. $A \vee C$.
3. $C \rightarrow D$.

$\therefore$ $B \vee D$.

4. $(A \rightarrow B) \& (C \rightarrow D)$. 1, 3; Conj.
5. $B \vee D$. 4, 2; C.D.

Natural Deduction

Of course, the real goal with natural deduction is to be able to take arguments in English, translate them into the language of logic, and then formally prove their validity.

Argument #3

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct this argument's formal proof of validity. This proof can be done in only *two* steps.

Layli being present is sufficient for **Majnun** being happy. **Nada** being pleased is necessary for **Layli** being present and **Majnun** being happy. Therefore, **Layli** is present only if **Nada** is pleased. (L, M, N)

Argument #3 (Solution)

Layli being present is sufficient for **Majnun** being happy. **Nada** being pleased is necessary for **Layli** being present and **Majnun** being happy. Therefore, **Layli** is present only if **Nada** is pleased. (L, M, N)

1.	$L \rightarrow M.$	
2.	$(L \,\&\, M) \rightarrow N.$	
∴	$L \rightarrow N.$	
3.	$L \rightarrow (L \,\&\, M).$	1, Abs.
4.	$L \rightarrow N.$	3, 2; H.S.

Next Class…

We will look at arguments that require even longer proofs of validity. Don't panic!