CRITICAL THINKING Lecture #14

Understanding Proofs by Natural Deduction

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The Nine Rules of Inference

I. Modus Ponens (M.P.)

2. *Modus Tollens* (M.T.)

I.
$$p \rightarrow q$$
.
 I. $p \rightarrow q$.

 2. p .
 $2. \sim q$.

 $\therefore q$.
 $2. \sim q$.

 $\therefore -p$.

4. Disjunctive Syllogism (D.S.)

I. $(p \rightarrow q) \& (r \rightarrow s)$. I. $p \lor q$. 2. $p \lor r$. 2. ~p. :. q. $\therefore q \lor s.$

7. Simplification (Simp.)

$$\frac{1. p \& q.}{\therefore p.}$$

8. Conjunction (Conj.)

I.
$$p$$
.
2. q .
 $\therefore p \& q$.

3. Hypothetical Syllogism (H.S.)

I.
$$p \rightarrow q$$
.
2. $q \rightarrow r$.
 $\therefore p \rightarrow r$.

5. Constructive Dilemma (C.D.)

$$\begin{array}{ccc} \mathbf{I}. & p \rightarrow q. \\ \hline \ddots & p \rightarrow (p \& q). \end{array} \end{array}$$

9. Addition (Add.)

$$\frac{\text{I.} \quad p.}{\therefore \quad p \lor q.}$$

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Natural Deduction: Instructions

Proving the validity of an argument using natural deduction works as follows:

- Translate the argument (if it is in English) into the language of symbolic logic, I.
- Put the argument into argumentative form, and 2.
- Use the nine rules of inference to derive the conclusion from the premises. 3.



Longer Formal Proofs

So far we have just seen arguments whose arguments only require *one* step in order to construct a proof of validity via natural deduction. That is, all that was needed was to recognize the pattern being used. However, most arguments will require proofs with more steps. Before diving into these, it is good to get a basic sense of how longer proofs work.

A Long Argument

Recall that last class I showed you the following formal proof of validity:



Now you should be in a better position to understand this proof as it is laid out above.



Understanding Formal Proofs

In starting to practice natural deduction, it is useful to begin by looking at correct formal proofs of validity, but with the explanation of each step left blank. We then fill in these blanks in the proof by trying to recognize which rule of inference can be used to get us to that step.





Proof #1

Fill in the blanks for the following proof of validity:



Proof #1 (Solution)

Fill in the blanks for the following proof of validity:



1; Simp.

2, 4; M.P.

3, 5; Conj.



Proof #2

Fill in the blanks for the following proof of validity:





Proof #2 (Solution)

Fill in the blanks for the following proof of validity:



1; Simp.

2, I; C.D.

5, 3; D.S.



Proof #3

Fill in the blanks for the following proof of validity:



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Proof #3 (Solution)

Fill in the blanks for the following proof of validity:



1, 5; Conj. 6, 7; C.D.





THREE WAYS TO LEARN NATURAL DEDUCTION 1. PRACTICE, 2. **PRACTICE**, 3. PRACTICE.

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Next Class...

We will do a workshop on doing simple formal proofs of validity using natural deduction. This will give you practice in recognizing the valid rules of inference and get you more comfortable with formal proofs.







References

[Blank t-shirt template]. (n.d.). [Digital image]. Retrieved March 3, 2017 from http://www. clipartbest.com/cliparts/pi5/GxA/pi5GxA5iB.jpeg.