CRITICAL THINKING

Lecture #9

Symbolic Logic & Natural Language

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Translating English to Logic: *Instructions*

Translating English to the language of symbolic logic works as follows:

1. Use capital letters to label each simple positive statement involved (sometimes these capital letters may be provided for you, sometimes they may not),

2. Perform statement classification (recall this from the first week of class),

3. Combine those capital letters with the logical operators to symbolize the results of statement classification, and

4. Be sure to use the grouping punctuation (parentheses and/or brackets) as needed.

The "Alphabet" of Symbolic Logic

Specific **positive simple statements** are represented by upper-case, upright letters, like these:

$$\mathrm{A},\ \mathrm{B},\ \mathrm{C},\ \mathrm{D},\ \ldots\ ,\ \mathrm{Z}.$$

Generic **statements** (that is, statements that could be anything: positive, negative, compound, or any combination thereof) are represented by lower-case, italic letters, like these:

$$p,\ q,\ r,\ \ldots\ ,\ z.$$

Four **logical operators/connectives** are represented as follows:

$$\&,\ \sim,\ \vee,\ \rightarrow.$$

Grouping punctuation is represented (in a manner similar to mathematics) as follows:

$$(,\ ),\ [,\ ],\ \{,\ \}.$$

Conjunction

Recall that a **conjunctive statement** asserts the truth of *all* its statements. It is symbolized using & (called "ampersand").

So the conjunctive statement $p \mathbin{\&} q$ asserts that statements p and q are both true. In this example, p and q are the **conjuncts**.

Note: By using the lower-case, italic letters p and q, this means that *any* two generic statements can be connected together as the conjuncts within a conjunctive statement.

Conjunction: *Example*

Consider the following conjunctive statement:

Logic is fun **and** logic is hard.

The conjuncts are simple positive statements, which are symbolized:

F: Logic is **fun**.
H: Logic is **hard**.

The entire conjunctive statement is then symbolized as F & H.

Note: We are *now* using the upper-case, upright letters F and H because each represents a specific, simple positive statement.

Conjunction: *Further Examples*

As we saw before, there are a lot of *other* ways to express the *exact same logic* symbolized by F & H:

Logic is fun **and** hard.

Logic is **both** fun **and** hard.

Logic is fun, **also** it is hard.

Logic is fun **but** hard.

Logic is fun, **yet** it is hard.

Logic is fun, **though** it is hard.

These certainly have different *connotations*, but they all have the same *logical* content.

Negation

Recall that a **negative statement** asserts that a given statement is false. It is symbolized using ~ (called "tilde").

So the negative statement $\sim p$ asserts that statement p is false.

Note: Once more, the use of a lower-case, italic letter, p, means that now *any* generic statement can be negated. This is new, because we used to only considered negative simple statements. Now we can consider all sorts of negative statements, both simple *and* compound.

Negation: *Example*

Consider the following negative statement:

Logic is **not** an easy class.

This is made up of one simple positive statement, which is symbolized:

E: Logic is an **easy** class.

So the entire negated statement is symbolized as ~E.

Note: Once again, we use an upper-case, upright letter, E, to represent a specific, simple positive statement.

Negation: *Further Examples*

Not surprisingly, there are a lot of *other* ways to express the *exact same logic* symbolized by ~E:

It is false that logic is an easy class.

It is not the case that logic is an easy class.

It is not true that logic is an easy class.

Disjunction

Recall that a **disjunctive statement** asserts the truth of *at least one* of its statements. It is symbolized using $\vee$ (called "wedge").

So the disjunctive statement $p \vee q$ asserts that at least one of statements p and q is true. In this example, p and q are the **disjuncts**.

Note: Yet again, the use of the lower-case, italic letters p and q means that *any* two generic statements can be connected together as the disjuncts within a disjunctive statement.

Disjunction: *Example*

Consider the following disjunctive statement:

Logic is fun **or** logic is hard.

The disjuncts are simple positive statements, which are symbolized:

F: Logic is **fun**.
H: Logic is **hard**.

The entire conjunctive statement is then symbolized as $\mathsf{F} \vee \mathsf{H}$.

Note: Yes, we are again using those upper-case, upright letters F and H to represent the specific, simple positive statements involved.

Disjunction: *Further Examples*

As we have also already seen before, there are a lot of *other* ways to express the *exact same logic* symbolized by F ∨ H:

Logic is fun **or** hard.

Logic is **either** fun **or** hard.

Logic is fun **unless** it is hard.

As before, these may have different connotations, but they are all logically identical.

Disjunction: *Inclusive vs. Exclusive*

The word 'or' (and especially the word 'unless') can be used in two slightly different, but logically significant, ways.

Logic is fun **or** hard.

This is probably best described as *inclusive* **disjunction**, where the claim is that *at least one* of the disjuncts is true. Notice that this claim is still true when logic is both fun and hard. This is the type of disjunction represented by $\vee$.

So the above statement is best symbolized as $F \vee H$.

Disjunction: *Inclusive vs. Exclusive*

However, 'or' (and 'unless') can be used in a logically different way:

I will pass **or** fail logic.

This is *exclusive* **disjunction**, where the claim is that *exactly one* of the disjuncts is true. So this statement is more precisely stated as:

I will pass **or** fail logic, **but not both**.

This is then symbolized differently:

$$(P \vee F) \,\&\, {\sim}(P \,\&\, F).$$

or but not both

Implication

Recall that a **hypothetical statement** has the form of "if … then …", asserting that whenever the "if" part is true, the "then" part must be true as well. It is symbolized using $\rightarrow$ (called "arrow").

So the hypothetical statement $p \rightarrow q$ asserts that if statement p is true, then statement q is true. In this example, p is the **antecedent** and q is the **consequent**.

Note: As you have probably figured out, the use of the lower-case, italic letters p and q means that *any* two generic statements can be connected together as antecedent and consequent within a hypothetical statement.

Implication: *Example*

Consider the following disjunctive statement:

If I study hard **then** I pass the class.

Both antecedent and consequent are simple positive statements, which are symbolized:

S: I **study** hard.
P: I **pass** the class.

The entire conjunctive statement is then symbolized as $\mathrm{S} \rightarrow \mathrm{P}$.

Note: Do I need to mention those upper-case, upright letters S and P?

Implication: *Further Examples*

Now there are a lot of *other* ways to express the *exact same logic* symbolized by $S \rightarrow P$:

If I study hard I pass the class.

My studying hard will **cause** me to pass the class.

I pass the class **if** I study hard.

Passing the class is a **necessary condition** for studying hard.

I study hard **only if** I pass the class.

Studying hard is a **sufficient condition** for passing the class.

These may be the tricky ones to remember!

Implication: *Necessary Conditions*

Consider the following hypothetical statement:

Taking quiz #4 is a ***necessary* condition** for getting an A on it.

The claim here is that taking quiz #4 is *required* to get an A on it. (Obviously, if you do not take the quiz, then you cannot get an A on it.) However, simply taking the quiz is *not enough* to get A on it. (Obviously, you must earn a certain number of points to get the A.) Even so, when someone gets an A on quiz #4, you know for sure that they took the quiz.

So the idea is that *if* you get an A on quiz #4 (A), *then* you have taken that quiz (T), which we symbolize as $A \rightarrow T$.

Implication: *Sufficient Conditions*

Consider the following hypothetical statement:

Six absences is a ***sufficient* condition** for failing critical thinking.

The claim here is that six absences are *enough* to fail the class. (This is stated in the course syllabus.) However, six absences is *not required* to fail the class. (Obviously, it is possible to fail the course in *other* ways, for instance, by not taking any of the quizzes or exams.) In other words, just because someone fails the class, you may not know the specific reason for why they failed the class.

So the idea is that *if* you get six absences in the course (S), *then* you fail the class (F), which we symbolize as $S \rightarrow F$.

Implication: *Necessary vs. Sufficient Conditions*

Notice that "p is a ***necessary* condition** for q" is symbolized as $q \rightarrow p$. Necessary means that p is *required* (but may not be enough) to get q. The idea is that if you have q, then p was required to get it. So if q then p, or $q \rightarrow p$.

However, "p is a ***sufficient* condition** for q" is symbolized as $p \rightarrow q$. Sufficiency means that p is *enough* (but may not be required) to get q. The idea is that if you have p, then that is enough to get q. So if p then q, or $p \rightarrow q$.

Implication: *If vs. Only If*

An "if" without an "only" does what it usually does and indicates that what follows is the *antecedent* of a hypothetical. In this way, it functions like the phrase "is a necessary condition for".

You took quiz #4 **if** you got an A on it. ($A \rightarrow T$.)

You fail critical thinking **if** you have six absences. ($S \rightarrow F$.)

However, "only if", indicates that what follows is the *consequent* of a hypothetical. As such, it is like the phrase "is a sufficient condition for".

You can get an A on quiz #4 **only if** you take it. ($A \rightarrow T$.)

You can take six absences **only if** you want to fail. ($S \rightarrow F$.)

Appendix: Compound Statement Indicator Words

Common Conjunctive Indicators

and	but	while
both … and …	yet	however
also	though	furthermore

Common Disjunctive Indicators

or	either … or …	unless

Common Hypothetical Indicators

if … then …	if [vs.] only if	necessary [vs.] sufficient

Next Class…

We will learn how take apart statements and assess their truth value by using truth tables.