Introduction to Logical Reasoning

Workshop #8: Natural Deduction (Solutions)

Part I: Each of the following problems presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. Each proof can be completed in just *two* steps. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

1. 1. $(W \vee X) \rightarrow Y$.
   2. W.

   $\therefore$ Y.

   3. $W \vee X$. 2; Add. Correct statement [2], line reference [1], and inference rule [2].
   4. Y. 1, 3; M.P. Correct statement [1], line references [3], and inference rule [2].

Following directions [1]. No other mistakes [1].

2. 1. $D \rightarrow E$.
   2. $(E \rightarrow F) \,\&\, (F \rightarrow D)$.

   $\therefore$ $D \rightarrow F$.

   3. $E \rightarrow F$. 2; Simp. Correct statement [2], line reference [1], and inference rule [2].
   4. $D \rightarrow F$. 1, 3; H.S. Correct statement [1], line references [3], and inference rule [2].

Following directions [1]. No other mistakes [1].

Part II: The following problem presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. This proof can be completed in just *three* steps. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

1. 1. $Q \rightarrow R$.
   2. $R \rightarrow S$.
   3. $\sim S$.

   $\therefore$ $\sim Q \,\&\, \sim R$.

   4. $\sim R$. 2, 3; M.T. Correct statement [2], line references [3], and inference rule [2].
   5. $\sim Q$. 1, 4; M.T. Correct statement [2], line references [3], and inference rule [2].
   6. $\sim Q \,\&\, \sim R$. 4, 5; Conj. Correct statement [1], line references [3], and inference rule [2].

Following directions [1]. No other mistakes [1].

Please continue ▶

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Part III: Each of the following problems presents a valid argument in English. For each, (1) translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct that argument's formal proof of validity. Each proof can be completed in no more than *four* steps, but I'm not saying the exact number.

1. Either the journalism students **love** logic, or the journalism students **study** hard only if the professor **quizzes** them on the material. But if the journalism students do not **love** logic, then the professor **quizzes** them on the material only if he wants them to **understand** the material. The journalism students do not **love** logic. Therefore, if the journalism students **study** hard then the professor wants them to **understand** the material. (L, S, Q, U)

1.	$L \vee (S \rightarrow Q)$.		[2]
2.	$\sim L \rightarrow (Q \rightarrow U)$.		[2]
3.	$\sim L$.		[2]
$\therefore$	$S \rightarrow U$.		[2]
4.	$S \rightarrow Q$.	1, 3; D.S.	Correct statement [2], line references [3], and inference rule [2].
5.	$Q \rightarrow U$.	2, 3; M.P.	Correct statement [2], line references [3], and inference rule [2].
6.	$S \rightarrow U$.	4, 5; H.S.	Correct statement [1], line references [3], and inference rule [2].

Argumentative form [1]. Following directions [1]. No other mistakes [1].

2. If the **journalism** students love logic, then the **business** students love logic. If both the **journalism** and **business** students love logic, then either the computer **science** or **communications** students love logic. If the computer **science** or **communications** students love logic, then the **professor** is not sad. If the **journalism** students loving logic is a sufficient condition for the **professor** not being sad, then the **dean** is pleased. As a result, the **dean** is pleased. (J, B, S, C, P, D)

1.	$J \rightarrow B$.		[2]
2.	$(J \,\&\, B) \rightarrow (S \vee C)$.		[2]
3.	$(S \vee C) \rightarrow \sim P$.		[2]
4.	$(J \rightarrow \sim P) \rightarrow D$.		[2]
$\therefore$	D.		[2]
5.	$J \rightarrow (J \,\&\, B)$.	1; Abs.	Correct statement [2], line references [1], and inference rule [2].
6.	$J \rightarrow (S \vee C)$.	5, 2; H.S.	Correct statement [2], line references [3], and inference rule [2].
7.	$J \rightarrow \sim P$.	6, 3; H.S.	Correct statement [2], line references [3], and inference rule [2].
8.	D.	4, 7; M.P.	Correct statement [1], line references [3], and inference rule [2].

Argumentative form [1]. Following directions [1]. No other mistakes [1].