Introduction to Logical Reasoning

Workshop #7: Identifying Valid Argument Forms (Solutions)

Part I: Each of the following problems presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. Each proof will only require *one* step, so essentially the task is to identify the one rule of inference that is enough to justify the conclusion from the stated premise(s).

1. 1. $X \rightarrow Y$.
   ∴ $X \rightarrow (X \& Y)$.
   2. $X \rightarrow (X \& Y)$. 1; Abs.

   Correct statement [1], line reference [1], and inference rule [2].

   Following directions [1]. No other mistakes [1].

2. 1. $(A \& B) \rightarrow C$.
   2. $\sim C$.
   ∴ $\sim(A \& B)$.
   3. $\sim(A \& B)$. 1, 2; M.T.

   Correct statement [1], line references [3], and inference rule [2].

   Following directions [1]. No other mistakes [1].

3. 1. $(W \vee X) \rightarrow \sim(W \rightarrow X)$.
   2. $W \vee X$.
   ∴ $\sim(W \rightarrow X)$.
   3. $\sim(W \rightarrow X)$. 1, 2; M.P.

   Correct statement [1], line references [3], and inference rule [2].

   Following directions [1]. No other mistakes [1].

4. 1. $\sim(M \rightarrow \sim N) \vee (Q \& R)$.
   2. $\sim\sim(M \rightarrow \sim N)$.
   ∴ $Q \& R$.
   3. $Q \& R$. 1, 2; D.S.

   Correct statement [1], line references [3], and inference rule [2].

   Following directions [1]. No other mistakes [1].

5. 1. $[(W \vee \sim X) \rightarrow D] \& (H \vee R)$.
   ∴ $(W \vee \sim X) \rightarrow D$.
   2. $(W \vee \sim X) \rightarrow D$. 1; Simp.

   Correct statement [1], line reference [1], and inference rule [2].

   Following directions [1]. No other mistakes [1].

6. 1. $(E \rightarrow F) \rightarrow G$.
   ∴ $(E \rightarrow F) \rightarrow [(E \rightarrow F) \& G]$.
   2. $(E \rightarrow F) \rightarrow [(E \rightarrow F) \& G]$. 1; Abs.

   Correct statement [1], line reference [1], and inference rule [2].

   Following directions [1]. No other mistakes [1].

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Part II: Each of the following problems presents a complete and correct formal proof of validity for an argument using natural deduction. For each proof, state the justification for each step within that proof (i.e., the numbered steps that start *after* the conclusion has been indicated by the ∴).

1. 1. $(A \vee B) \rightarrow C$.
   2. $(C \vee B) \rightarrow [A \rightarrow (D \vee E)]$.
   3. $A \,\&\, D$.

   ∴ $D \vee E$.

4. A.	3; Simp.	Correct line reference [1] and inference rule [2].
5. A ∨ B.	4; Add.	Correct line reference [1] and inference rule [2].
6. C.	1, 5; M.P.	Correct line references [3] and inference rule [2].
7. C ∨ B.	6; Add.	Correct line reference [1] and inference rule [2].
8. A → (D ∨ E).	2, 7; M.P.	Correct line references [3] and inference rule [2].
9. D ∨ E.	8, 4; M.P.	Correct line references [3] and inference rule [2].

Following directions [1]. No other mistakes [1].

2. 1. $W \rightarrow X$.
   2. $(W \rightarrow Y) \rightarrow (Z \vee X)$.
   3. $(W \,\&\, X) \rightarrow Y$.
   4. $\sim Z$.

   ∴ X.

5. W → (W & X).	1; Abs.	Correct line reference [1] and inference rule [2].
6. W → Y.	5, 3; H.S.	Correct line references [3] and inference rule [2].
7. Z ∨ X.	2, 6; M.P.	Correct line references [3] and inference rule [2].
8. X.	7, 4; D.S.	Correct line references [3] and inference rule [2].

Following directions [1]. No other mistakes [1].