Introduction to Logical Reasoning

Lecture #22

Further Categorical Inferences

Professor David Emmanuel Gray

Carnegie Mellon University in Qatar
Northwestern University in Qatar

Four Standard Forms of Categorical Statements

Universal Positive

A: All *X* is *Y*.

Shade in all of *X* not shared with *Y*.

Universal Negative

E: No *X* is *Y*.

Shade in all of *X* shared with *Y*.

Particular Positive

I: Some *X* is *Y*.

Dot-*x* in *X* shared with *Y*.

Particular Negative

O: Some *X* is not *Y*.

Dot-*x* in *X* not shared with *Y*.

Note: A complement like non-*S* or non-*P* can substitute *X* or *Y*.

Statement 1

Consider the following categorical statement:

> No students are lazy people.

Suppose that this statement is *true*. What can we then logically infer about the claim that "No lazy people are students"? Is it true, false, or its truth/falsity unknown?

Statement 2

Consider the following categorical statement:

> Some students are lazy people.

Suppose that this statement is *true*. What can we then logically infer about the claim that “Some lazy people are students”? Is it true, false, or its truth/falsity unknown?

Conversion

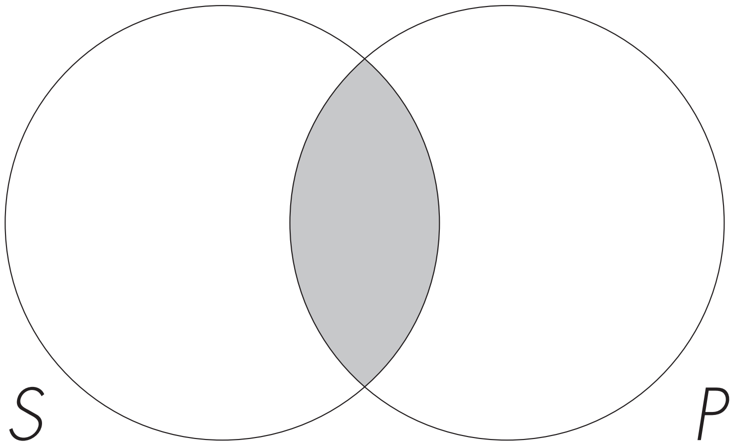
The **conversion** of a categorical statement swaps its subject (S) and predicate (P) terms to create a new categorical statement.

In some instances, the new statement will be logically equivalent to the original one. For example, the statement "No students are lazy people" (**E**: No S is P) is logically the same as "No lazy people are students" (**E**: No P is S).

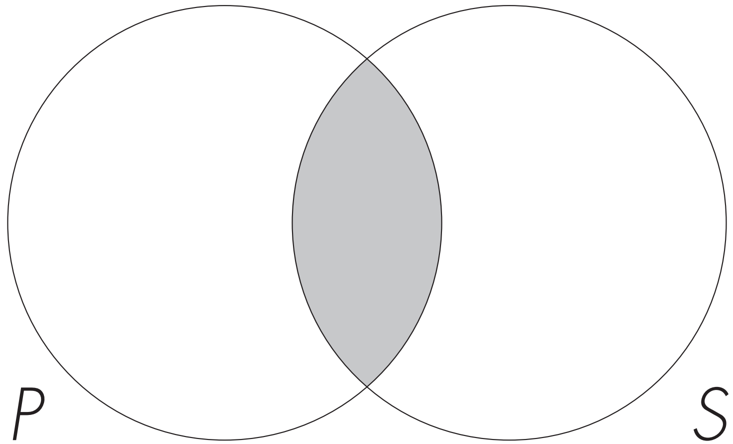
Note: To keep things constant, we fix the categories S and P using the first statement (S = students, P = lazy people), even though in the second statement S is the predicate term and P is the subject term.

Conversion

In general, any **E** statement and its conversion are logically the same.



E Statement: No *S* is *P*.

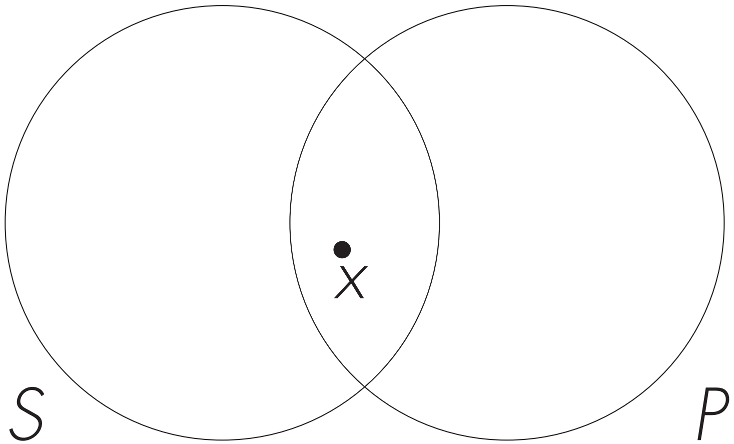


E's Conversion: No *P* is *S*.

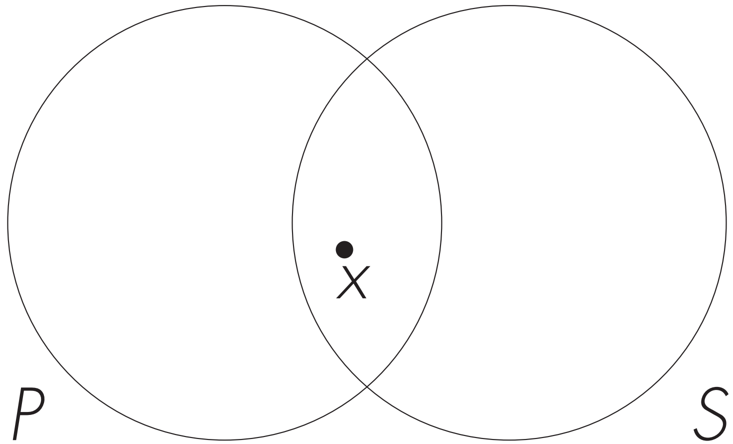
Note: Even though *S* and *P* are the same in both statements, the Venn diagram of the second statement (like all diagrams) has its left circle represent the statement's subject (now *P*) and its right circle represent the statement's predicate (now *S*).

Conversion

Similarly, any **I** statement and its conversion are logically the same.



I Statement: Some *S* is *P*.



I's Conversion: Some *P* is *S*.

Statement 3

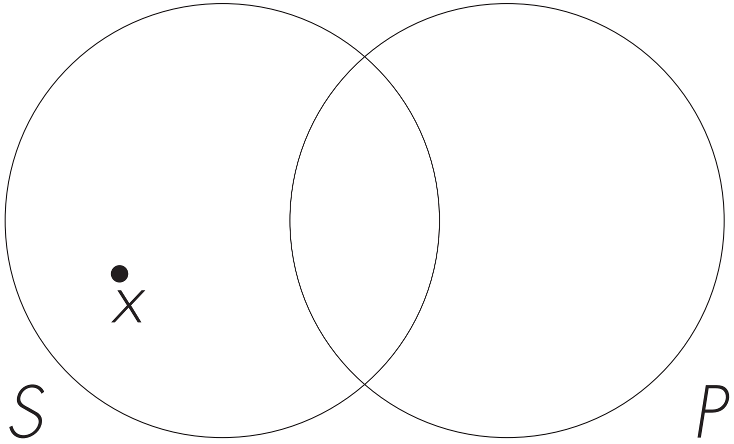
Consider the following categorical statement:

> Some students are not lazy people.

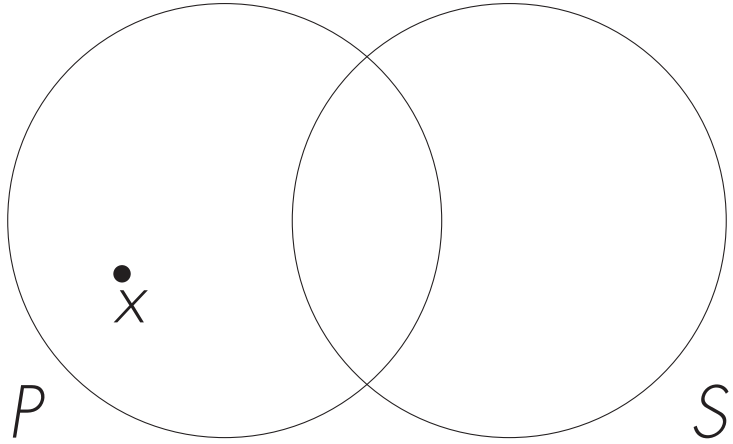
Suppose that this statement is *true*. What can we then logically infer about the claim that "Some lazy people are not students"? Is it true, false, or its truth/falsity unknown?

Conversion

In general, any **O** statement and its conversion are *not* logically the same.



O Statement: Some *S* is not *P*.

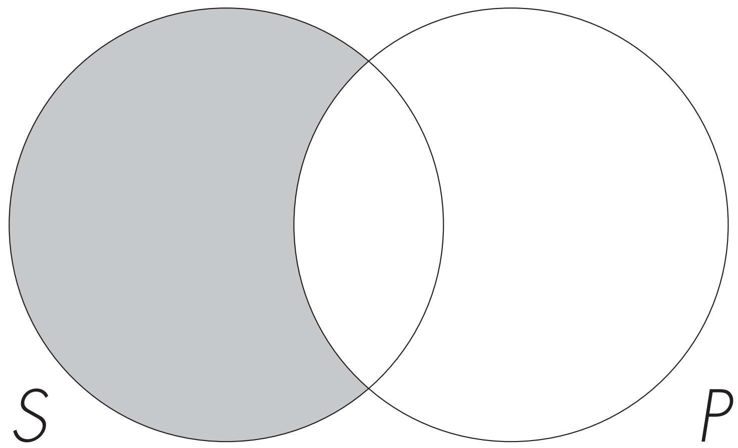


O's Conversion: Some *P* is not *S*.

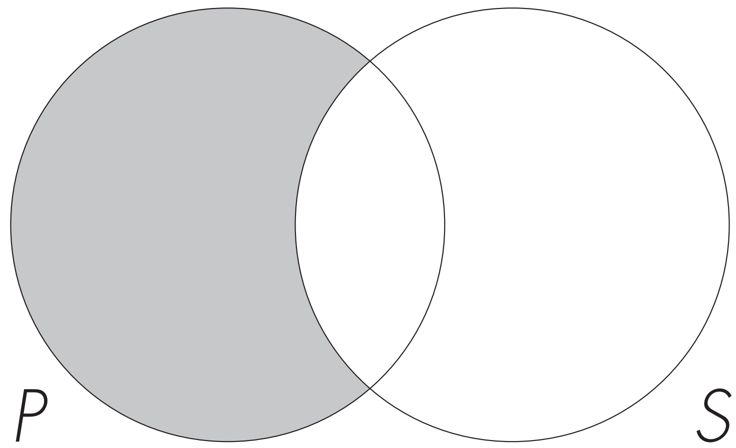
Look closely and you will see that the dox-*x* is actually *not* in the same place in both diagrams.

Conversion

Similarly, any **A** statement and its conversion are *not* logically the same.



A Statement: All *S* is *P*.



A's Conversion: All *P* is *S*.

Look closely and you will see that the shaded area is actually *not* the same in both diagrams.

Complement

Recall that for any subject (*S*) or predicate (*P*) term in a categorical statement, we may consider its **complement**. The complement of a category consists of everything *not* in that category. The complement of the subject term *S* is denoted as non-*S*; the complement of the predicate term *P* is denoted by non-*P*.

Statement 4

Consider the following categorical statement:

All students are lazy people.

Suppose that this statement is *true*. What can we then logically infer about the claim that "No students are non-lazy people"? Is it true, false, or its truth/falsity unknown?

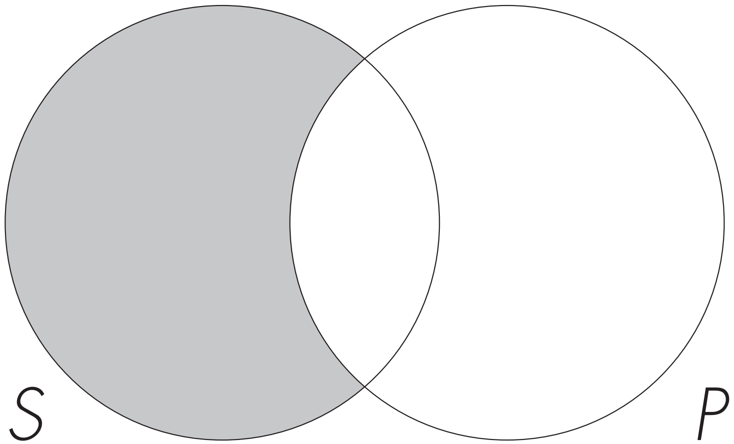
Obversion

The **obversion** of a categorical statement comes from flipping its quality and replacing the predicate (*P*) with that predicate's complement (non-*P*).

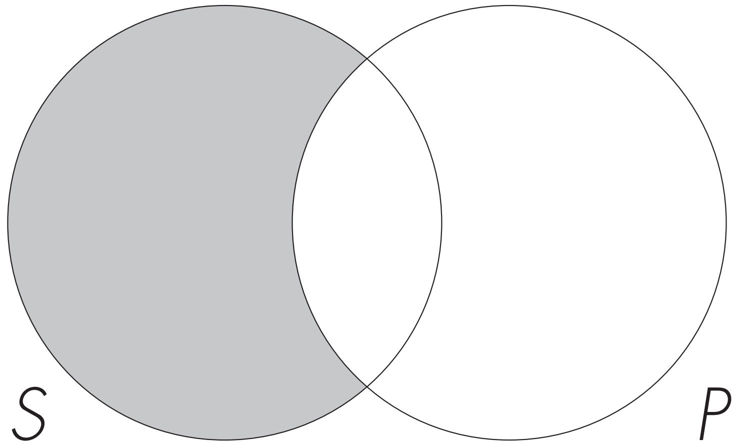
It turns out that the obversion of each of the standard four categorical statements is logically equivalent to the original statement. So, for instance, "All students are lazy people" (**A**: All *S* is *P*) is logically equivalent to its obversion: "No students are non-lazy people" (**E**: No *S* is non-*P*).

Obversion

In general, any **A** statement and its obversion (an **E** statement) are logically the same.



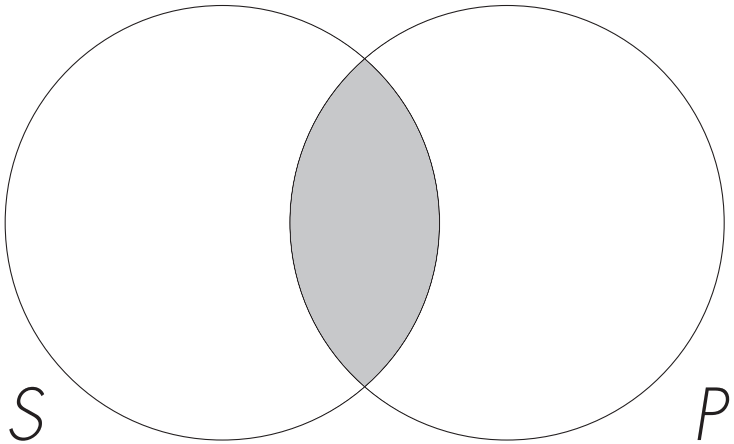
A Statement: All *S* is *P*.



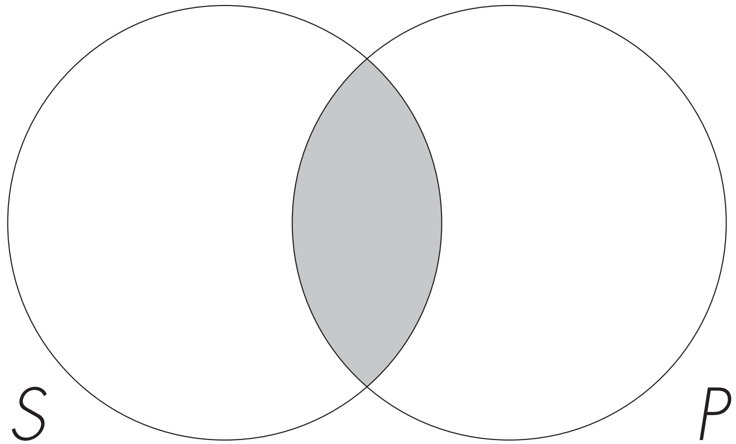
A's Obversion (**E** Statement): No *S* is non-*P*.

Obversion

Similarly, any **E** statement and its obversion (an **A** statement) are logically the same.



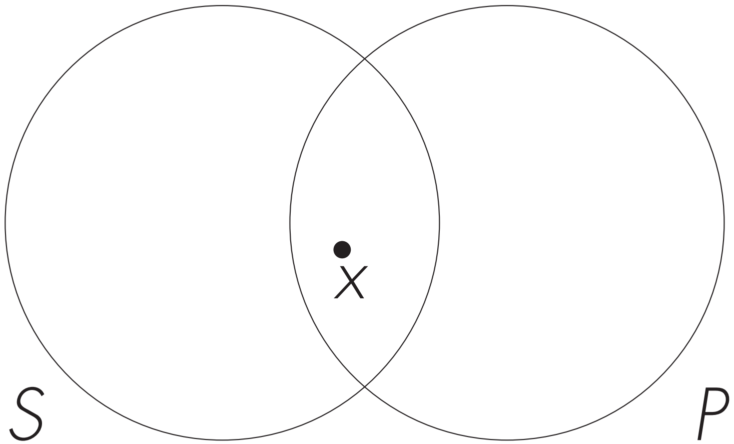
E Statement: No *S* is *P*.



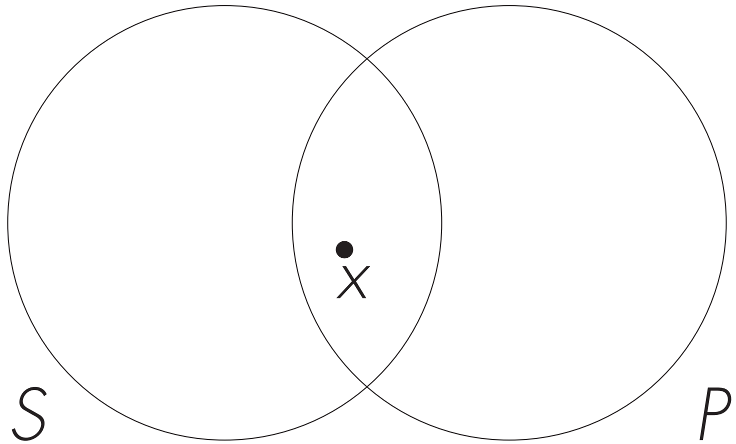
E's Obversion (**A** Statement): All *S* is non-*P*.

Obversion

And so for any **I** statement and its obversion (an **O** statement).



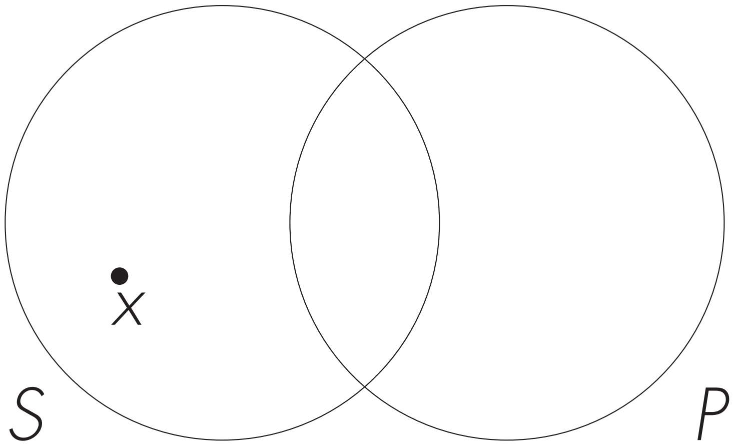
I Statement: Some *S* is *P*.



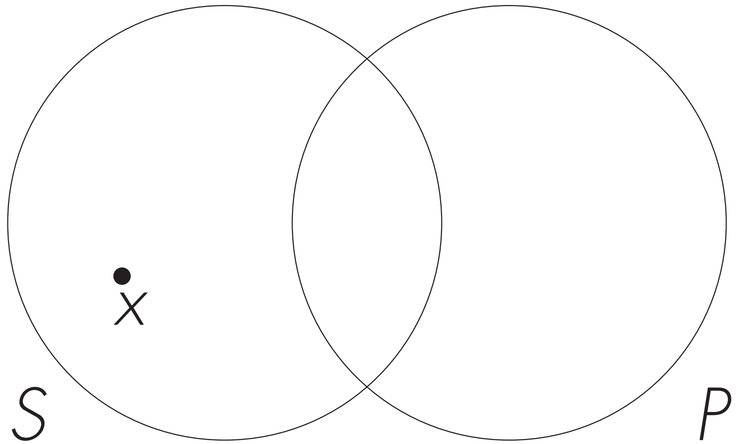
I's Obversion (**O** Statement): Some *S* is not non-*P*.

Obversion

And finally for any **O** statement and its obversion (an **I** statement).



O Statement: Some *S* is not *P*.



O's Obversion (**I** Statement): Some *S* is non-*P*.

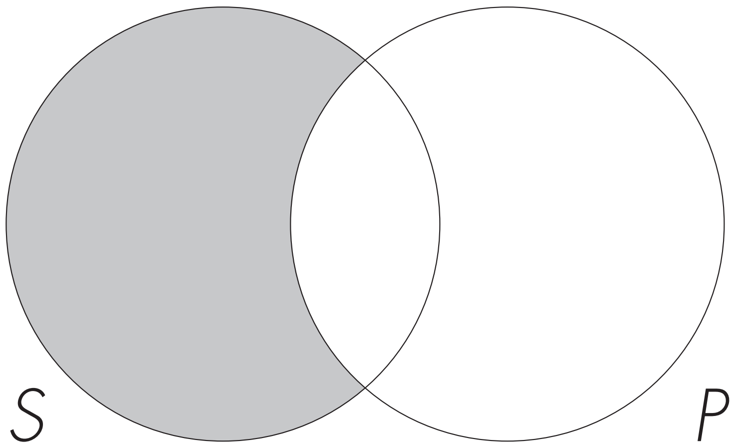
Contraposition

According to **contraposition**, a categorical statement is changed by (1) replacing its subject (*S*) term with that subject's complement (non-*S*), (2) replacing its predicate (*P*) term with that predicate's complement (non-*P*), and (3) swapping the new subject and new predicate.

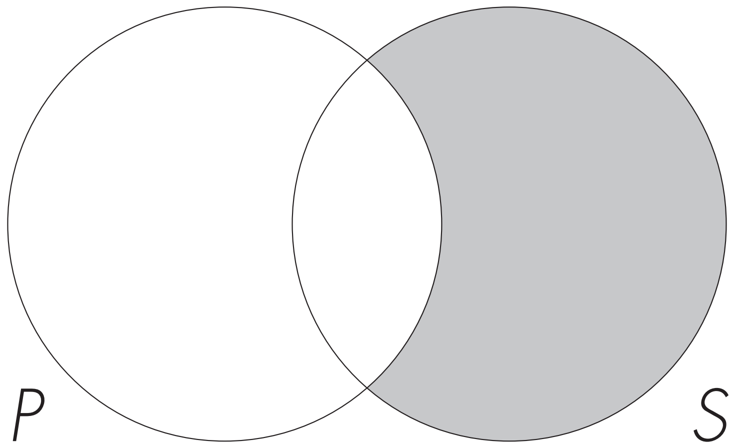
In *some* instances, the new statement will be logically equivalent to the original one. For example, the proposition "All students are lazy people" (**A**: All *S* is *P*) is logically the same as "All non-lazy people are non-students" (**A**: All non-*P* is non-*S*).

Contraposition

In general, any **A** statement and its contrapositive are logically the same.



A Statement: All *S* is *P*.

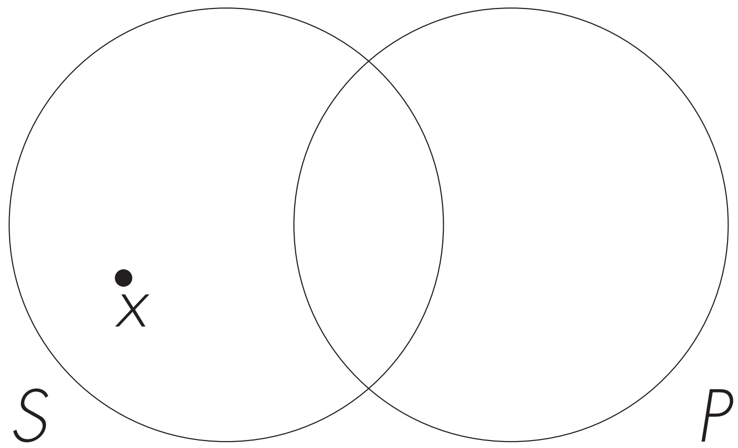


A's Contrapositive: All non-*P* is non-*S*.

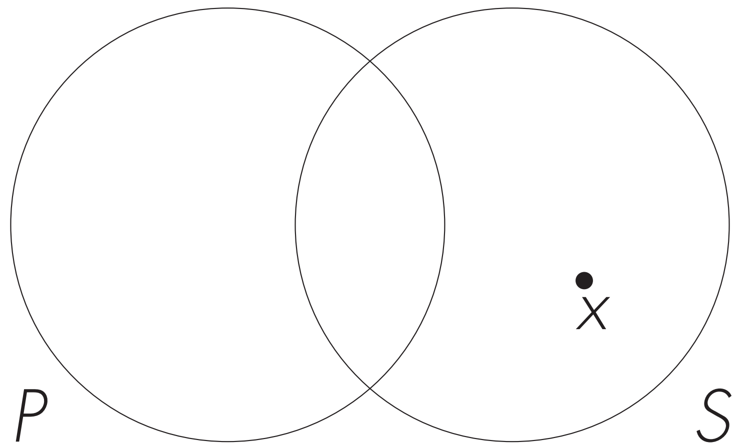
Look closely and you will see that the shaded area is actually the same in both diagrams.

Contraposition

Similarly, any **O** statement and its contrapositive are logically the same.



O Statement: Some *S* is not *P*.

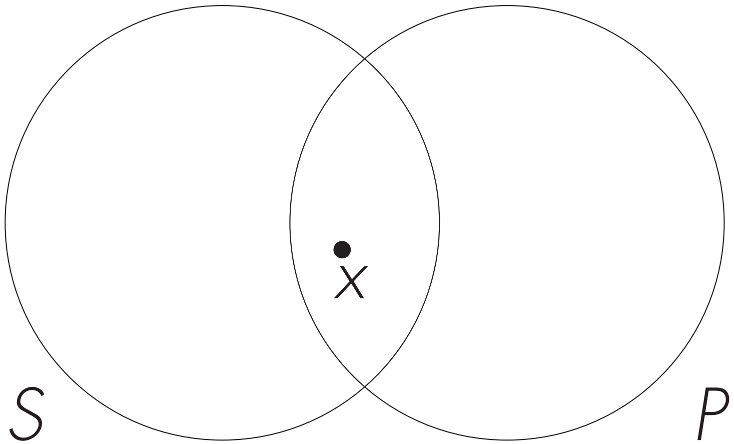


O's Contrapositive: Some non-*P* is not non-*S*.

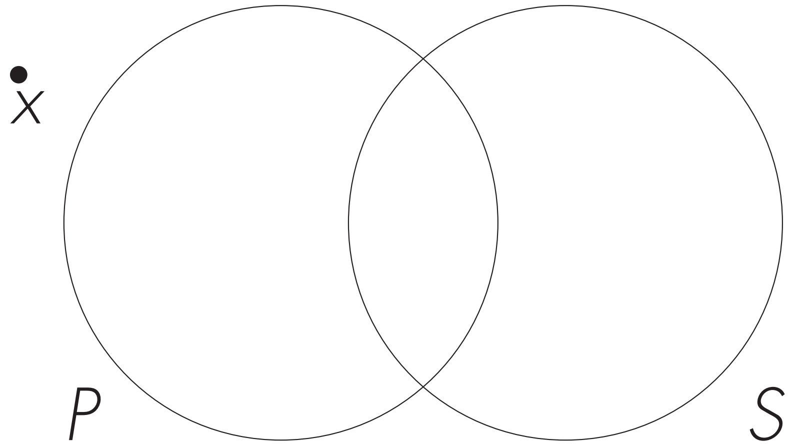
Look closely and you will see that the dox-*x* is actually in the same place in both diagrams.

Contraposition

However, any **I** statement and its contrapositive are *not* logically the same.



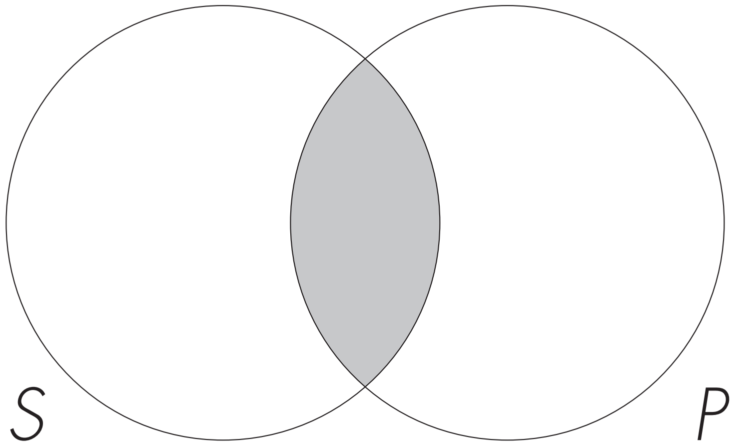
I Statement: Some *S* is *P*.



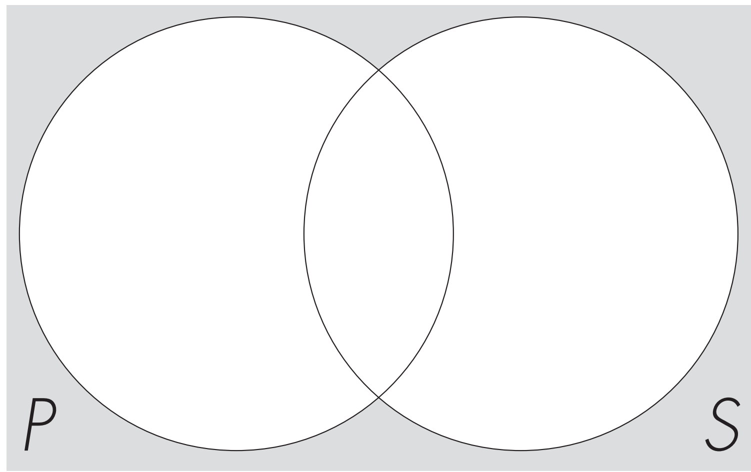
I's Contrapositive: Some non-*P* is non-*S*.

Contraposition

Similarly, any **E** statement and its contrapositive are *not* logically the same.



E Statement: No *S* is *P*.



E's Contrapositive: No non-*P* is non-*S*.

Categorical Inferences

Do not let all of this overwhelm you. Never forget: if you ever get lost, just make a Venn diagram.

From that simple diagram, you should be able assess any inference.


LOST?
MAKE A
Venn Diagram!

Next Class...

We will have a workshop on using Venn diagrams for making inferences from one categorical statement to another.

Also, please do not forget to turn in your response to the Lecture #22 Questionnaire on your way out.