Introduction to Logical Reasoning

Lecture #21

The Square of Opposition

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Four Standard Forms of Categorical Statements

Universal Positive

A: All *X* is *Y*.

Shade in all of *X* not shared with *Y*.

Universal Negative

E: No *X* is *Y*.

Shade in all of *X* shared with *Y*.

Particular Positive

I: Some *X* is *Y*.

Dot-*x* in *X* shared with *Y*.

Particular Negative

O: Some *X* is not *Y*.

Dot-*x* in *X* not shared with *Y*.

Note: A complement like non-*S* or non-*P* can substitute *X* or *Y*.

Statement 1

Consider the following categorical statement:

All students are hard workers.

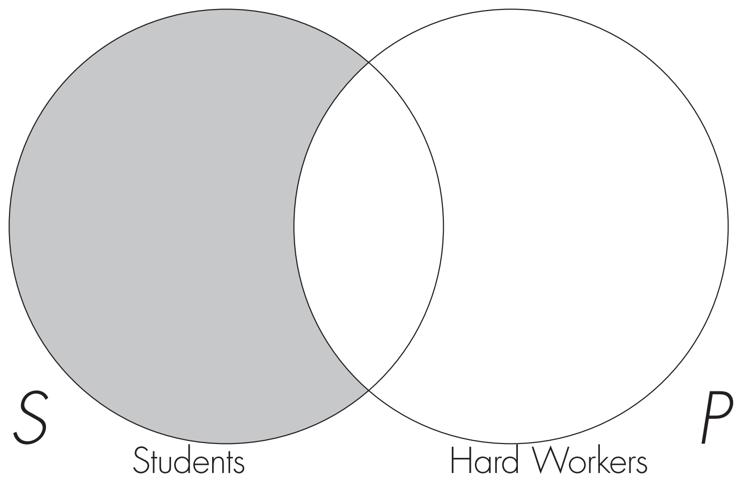
Suppose that this statement is *true*. What can we then logically infer about the claim that "Some students are not hard workers"? Is it true, false, or its truth/falsity unknown?

This is an example of making an inference from one categorical statement to a second one that involves the *same* subject (*S* = students) and predicate (*S* = hard workers) terms.

Categorical Inferences

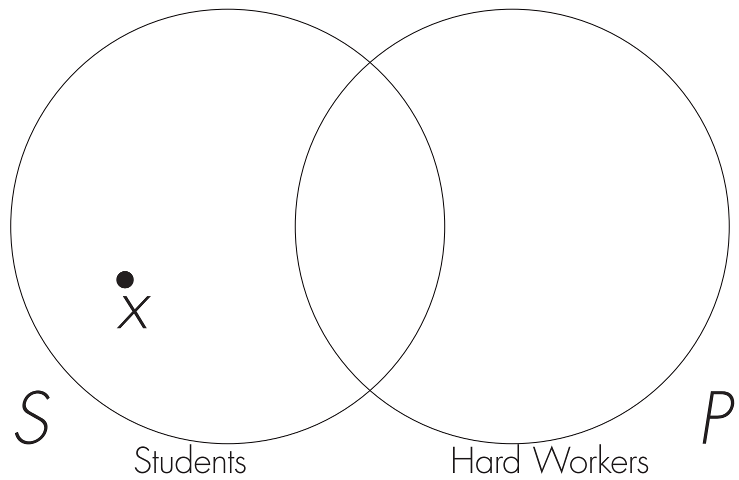
When making a categorical inference, it is easiest to first draw out the associated Venn diagrams for the categorical statements, and then compare the pictures.

All students are hard workers.



A Statement: All *S* is *P*.

Some students are not hard workers.



O Statement: Some *S* is not *P*.

True → **??**

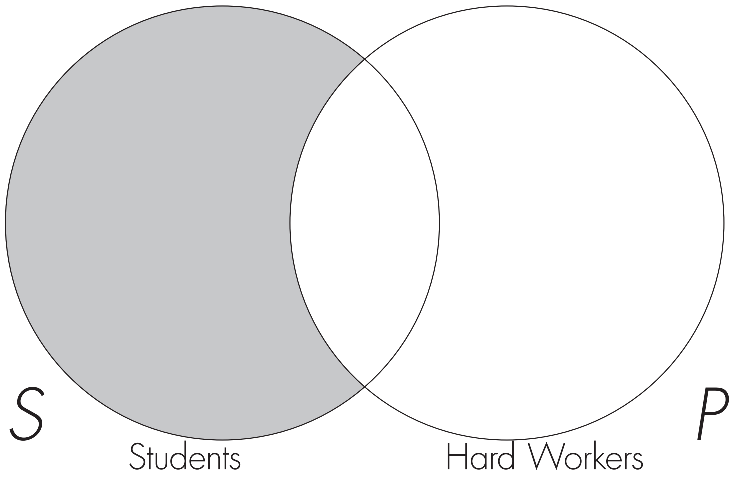
Categorical Inferences

The **A** statement (All *S* is *P*) claims there is *nothing* in the area of *S* outside of *P* (the **A** diagram shades that zone). Meanwhile, the **O** statement (Some *S* is not *P*) says there is *something* in that very same area (the **O** diagram has a dot-*x* there). This means that both statements cannot be true at the same time!

So given that "All students are hard workers" (**A**) is *true*, the claim that "Some students are not hard workers" (**O**) must be *false*.

Categorical Inferences

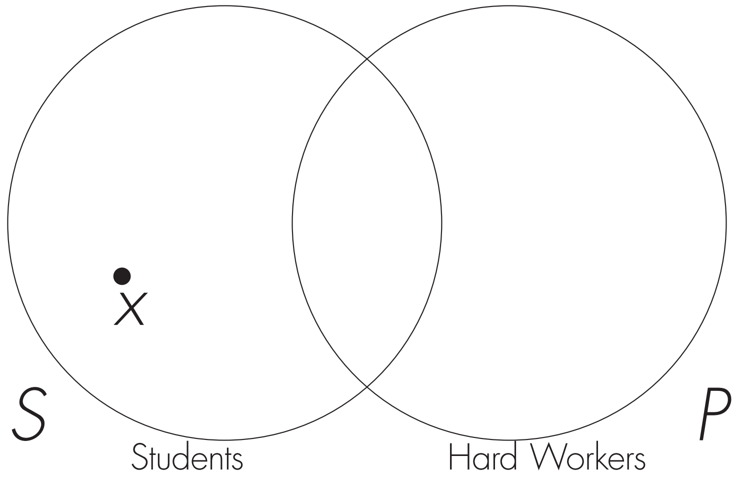
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.

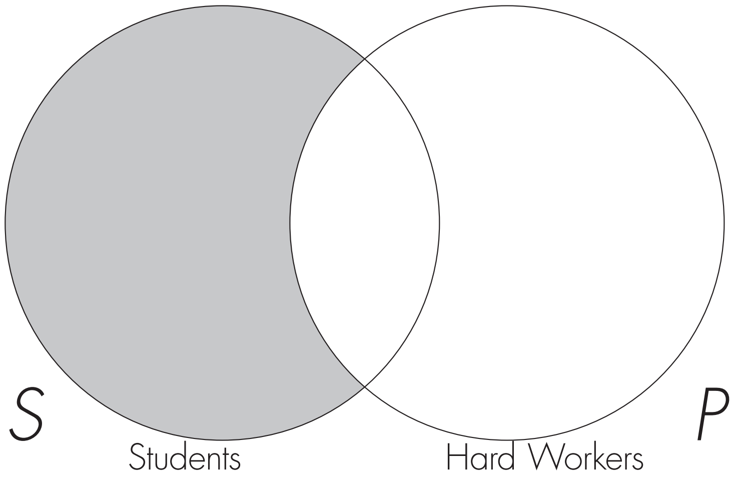


O Statement: Some *S* is not *P*.

??

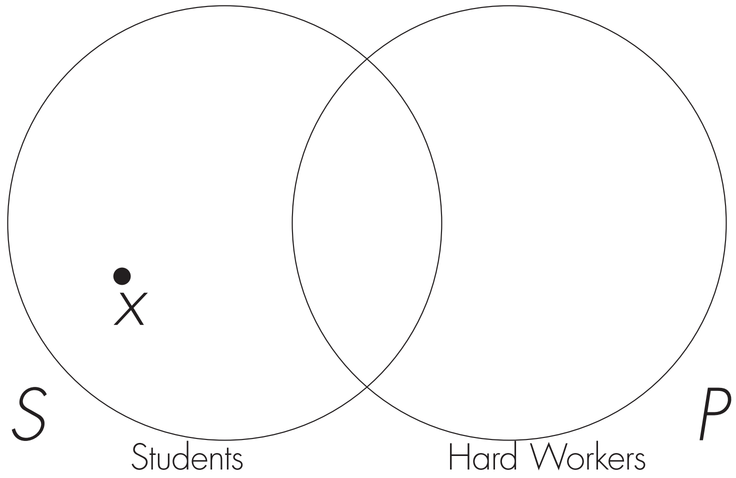
Categorical Inferences

All students are hard workers.



A Statement: All *S* is *P*.

Some students are not hard workers.



O Statement: Some *S* is not *P*.

True → **False!**

Statement 1

Consider the original categorical statement again:

All students are hard workers.

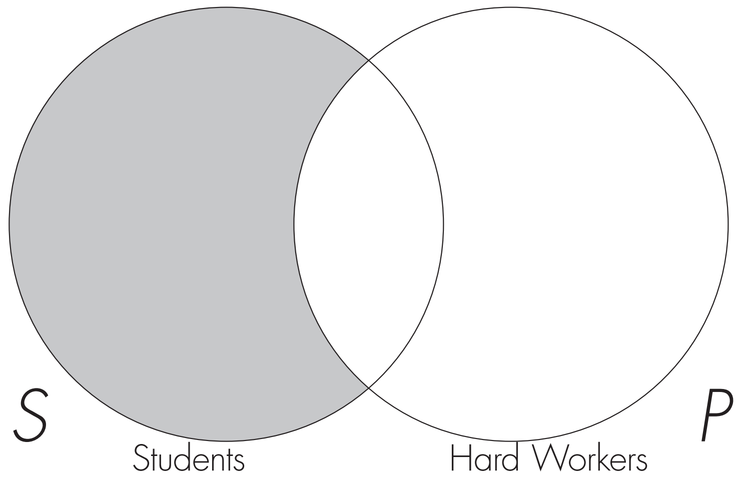
Now suppose that this statement is *false*. What can we then logically infer about the claim that "Some students are not hard workers"? Is it true, false, or its truth/falsity unknown?

This is an example of making an inference from one categorical statement to a second one that involves the *same* subject (*S* = students) and predicate (*S* = hard workers) terms.

Categorical Inferences

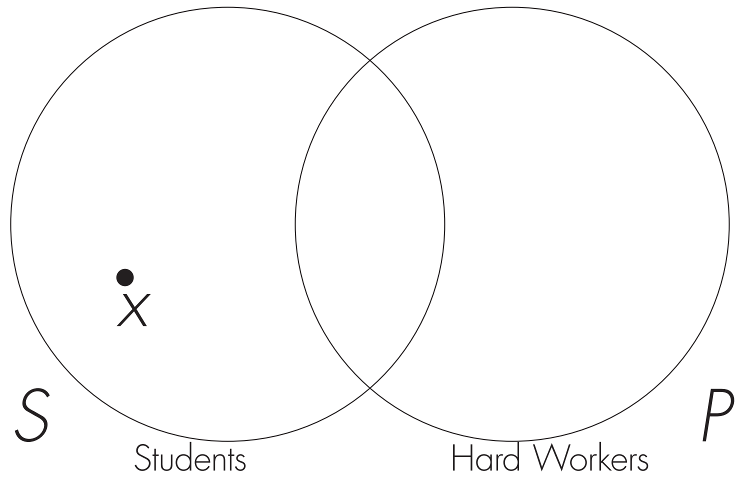
We need to draw the Venn diagrams, but how do we understand the Venn diagram for a false statement?

All students are hard workers.



A Statement: All *S* is *P*.

Some students are not hard workers.

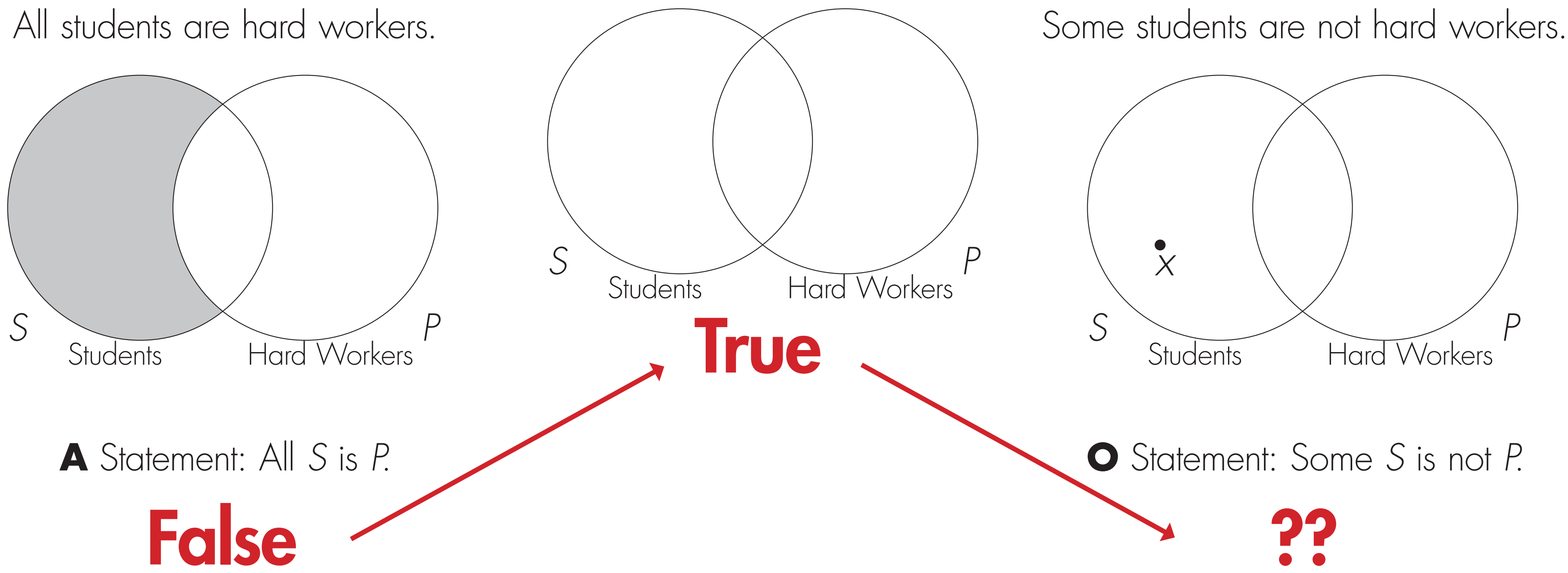


O Statement: Some *S* is not *P*.

False → **??**

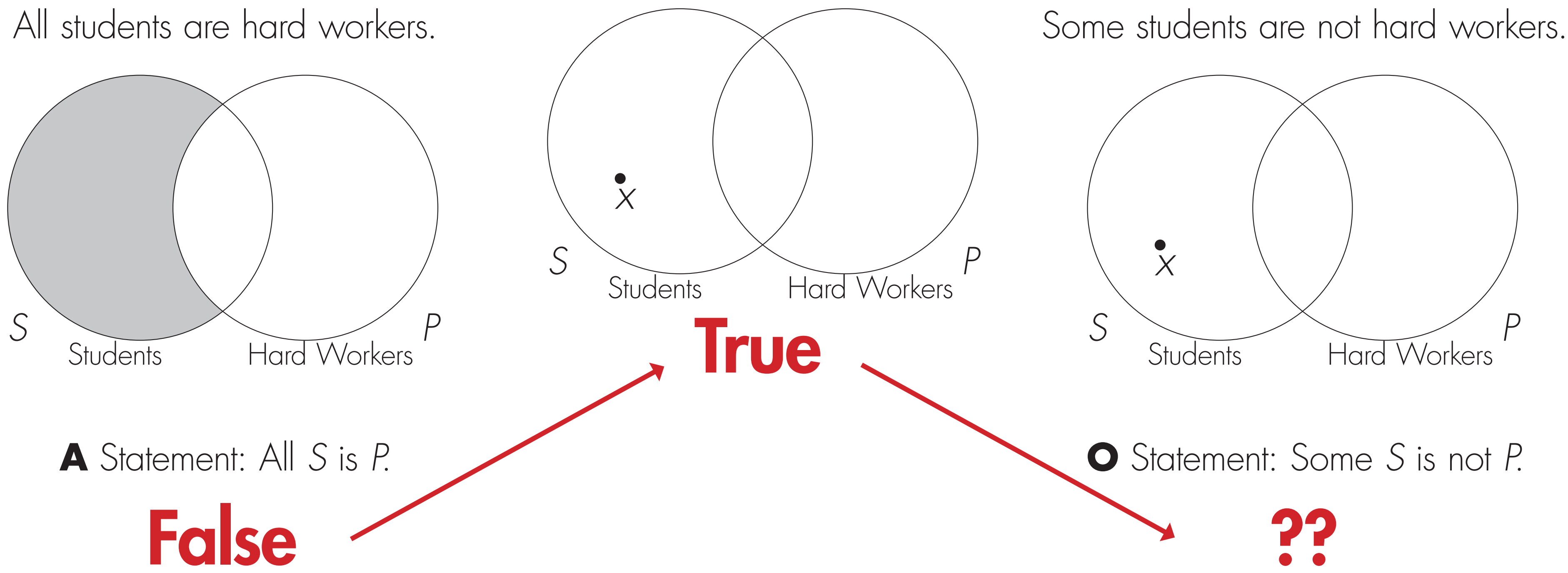
Categorical Inferences

We solve this by creating an intermediary Venn diagram that shows us what is truly going on.



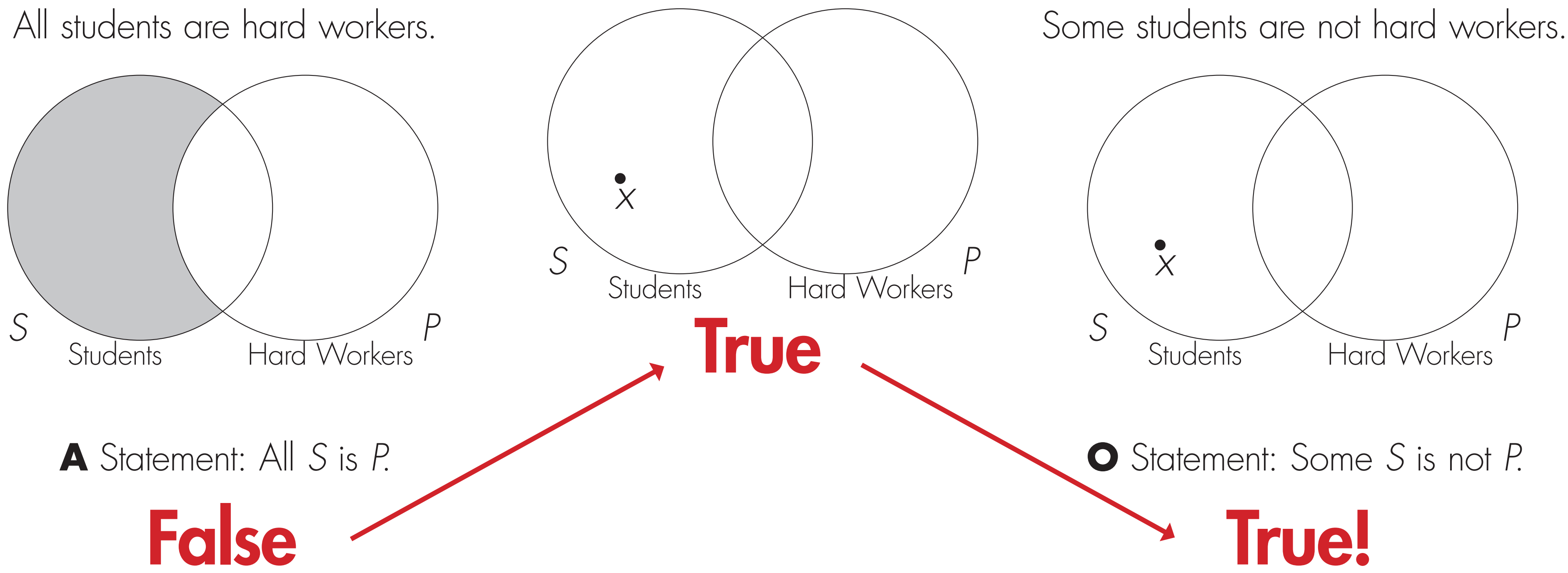
Categorical Inferences

Well, it is given that it is incorrect to claim the area of students outside of hard workers is empty. So there must be something there after all!



Categorical Inferences

Now we see quite clearly that the **O** statement must be true. After all, it has the exact same diagram that we know already to be true.



Statement 2

Consider the following categorical statement:

> No students are hard workers.

Suppose that this statement is *true*. What can we then logically infer about the claim that "Some students are hard workers"? Is it true, false, or its truth/falsity unknown?

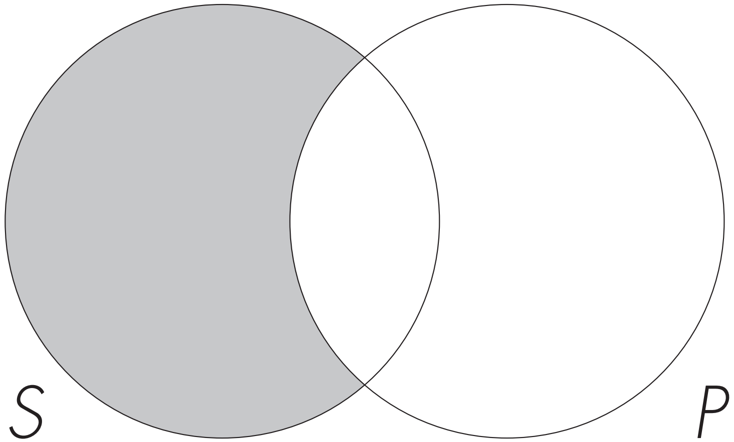
Contradictories

Two statements are **contradictories** if they both cannot be true at the same time and they both cannot be false at the same time.

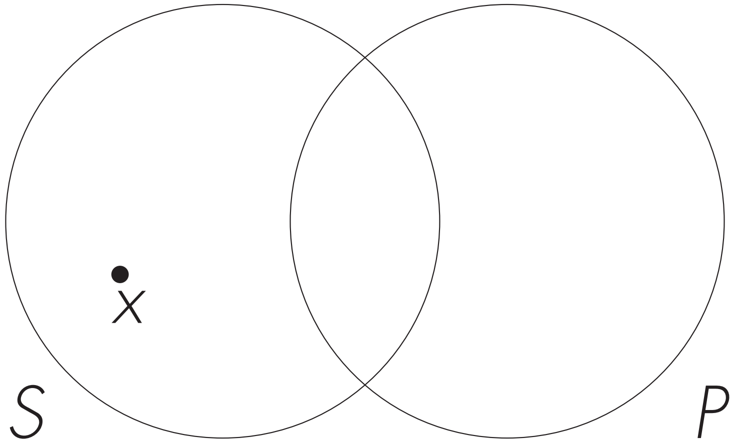
For instance, the statements "All students are hard workers" (**A**: All *S* is *P*) and "Some students are not hard workers" (**O**: Some *S* is not *P*) are contradictories. Both cannot be true and both cannot be false. So if you know one is true, the other must be false, and vice versa.

Contradictories

In general, **A** and **O** statements involving the same subject (*S*) and predicate (*P*) terms are always contradictories.



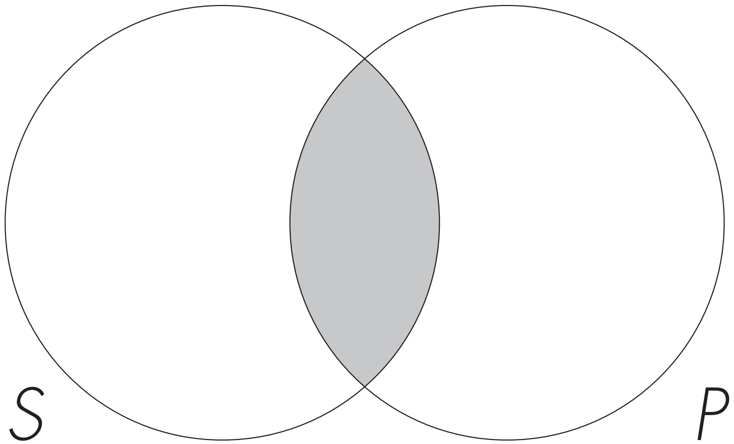
A Statement: All *S* is *P*.



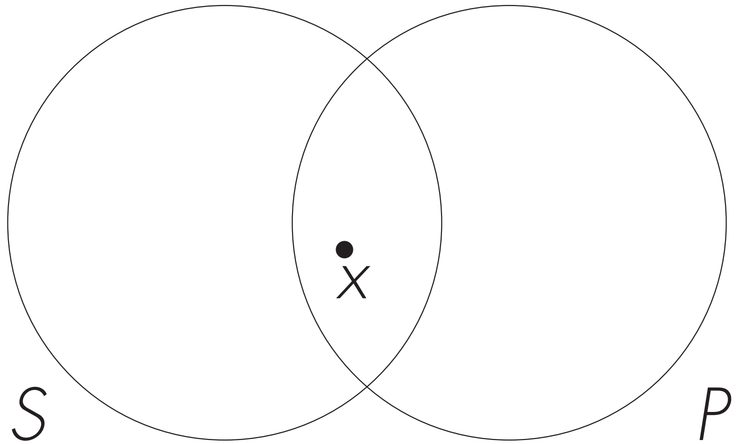
O Statement: Some *S* is not *P*.

Contradictories

Similarly, **I** and **O** statements involving the same subject (*S*) and predicate (*P*) terms are also always contradictories.



E Statement: No *S* is *P*.



I Statement: Some *S* is *P*.

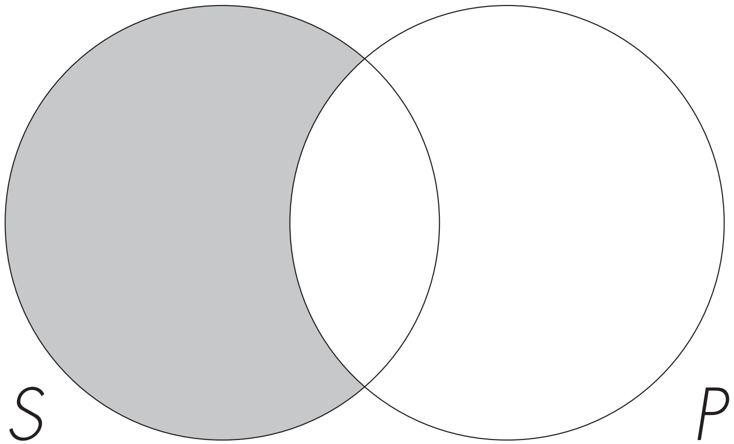
Contraries

Two statements are **contraries** if they both cannot be true, though both may be false.

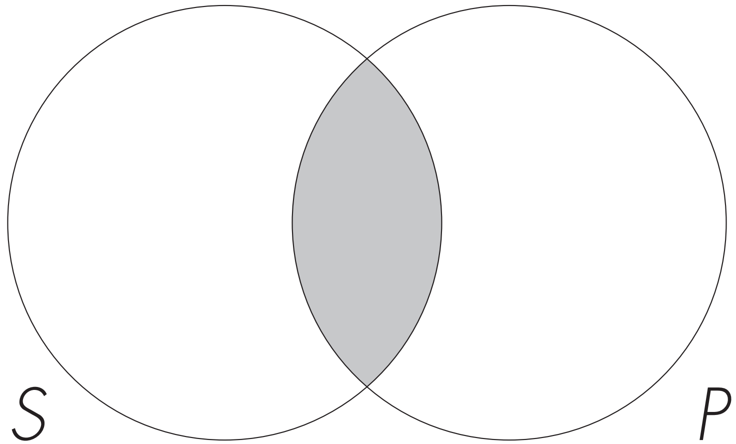
For instance, the statements "All students are hard workers" (**A**: All S is P) and "No students are hard workers" (**E**: No S is P) are contraries. Both cannot be true: if one is true, the other must be false. However, both statements could, in fact, be false: there might be some students who are hard workers and some others who are not.

Contraries

In general, **A** and **E** statements involving the same subject (*S*) and predicate (*P*) terms cannot both be true.*



A Statement: All *S* is *P*.

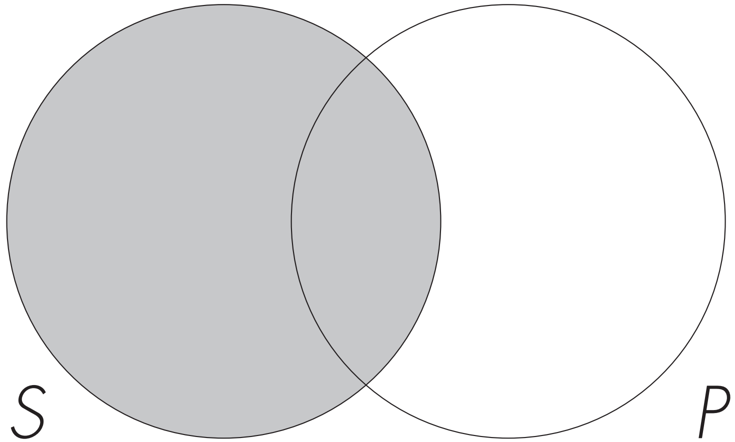


E Statement: No *S* is *P*.

***Note:** This only works when the subject term is not empty! Unless told otherwise, *always* assume the subject (*S*) term is not empty.

Contraries

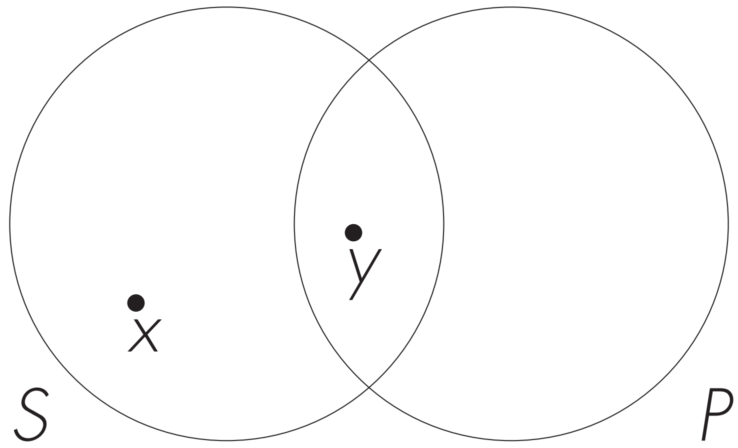
Both **A** and **E** statements cannot be true because that would mean the subject (*S*) term is empty.*



***Once again:** Unless told otherwise, *always* assume the subject (*S*) term is not empty.

Contraries

However, both **A** and **E** statements could be false. There is nothing problematic about that.



Therefore, **A** and **E** statements involving the same subject (*S*) and predicate (*P*) terms are contraries.

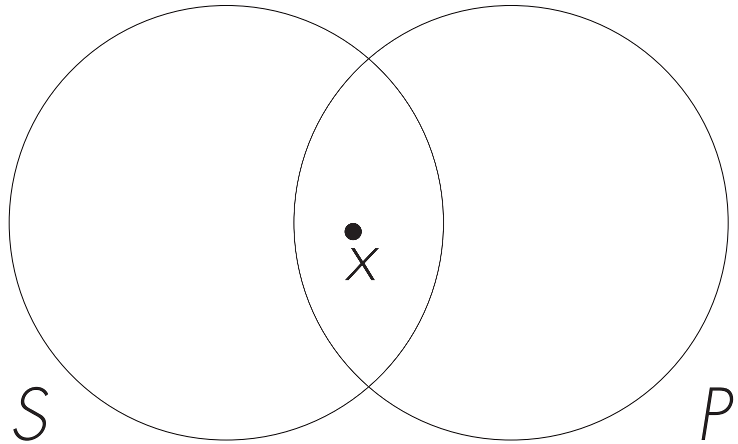
Subcontraries

Two statements are **subcontraries** if they both cannot be false, though they both may be true.

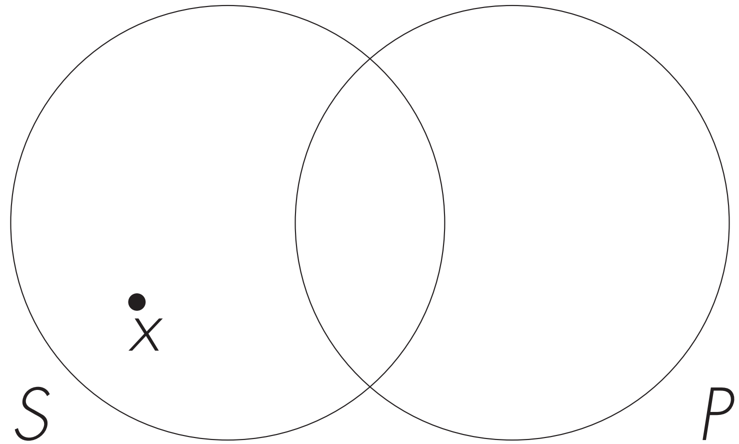
For instance, the statements "Some students are hard workers" (**I**: Some *S* is *P*) and "Some students are not hard workers" (**O**: Some *S* is not *P*) are subcontraries. Both cannot be false: if one is false, the other must be true. However, both statements could be true: as already noted, there might be some students who are hard workers and some others who are not.

Subcontraries

In general, **I** and **O** statements involving the same subject (*S*) and predicate (*P*) terms cannot both be false.*



I Statement: Some *S* is *P*.

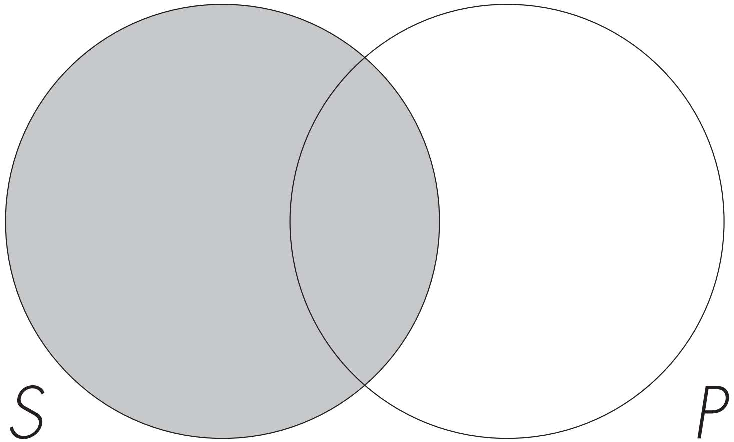


O Statement: Some *S* is not *P*.

***Note:** This also only works when the subject term is not empty!

Subcontraries

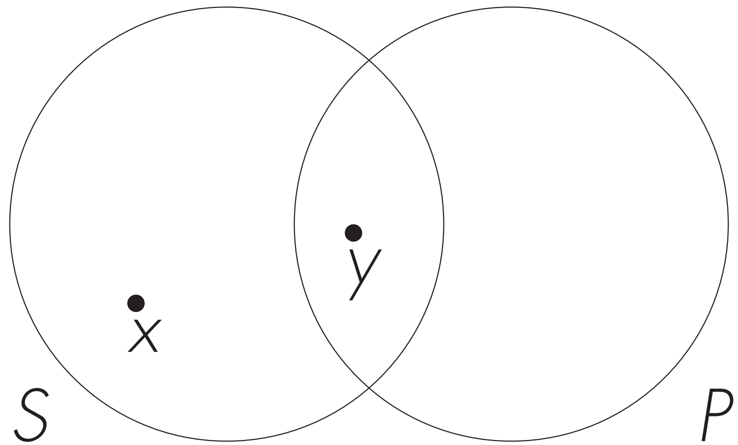
Both **I** and **O** statements cannot be false because that would mean the subject (*S*) term is empty.*



***Once again:** Unless told otherwise, *always* assume the subject (*S*) term is not empty.

Subcontraries

However, both **I** and **O** statements could be true. There is nothing problematic about that.



Therefore, **I** and **O** statements involving the same subject (S) and predicate (P) terms are subcontraries.

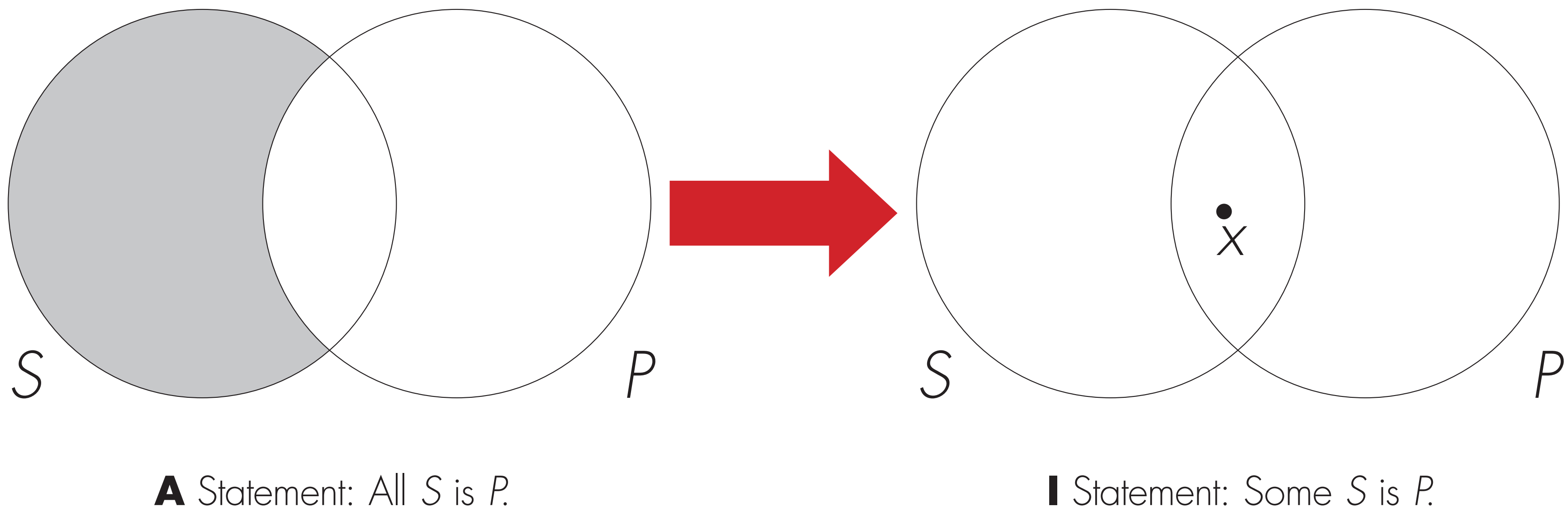
Subalternation

According to **subalternation**, any true *universal* categorical statement may be transformed into a true *particular* one. Going the other direction, subalternation says that any false particular categorical statement may be transformed into a false universal one.

So, for instance, if the statement "All students are hard workers" (**A**: All *S* is *P*) is true, then "Some students are hard workers" (**I**: Some *S* is *P*) is trivially true as well.

Subalternation

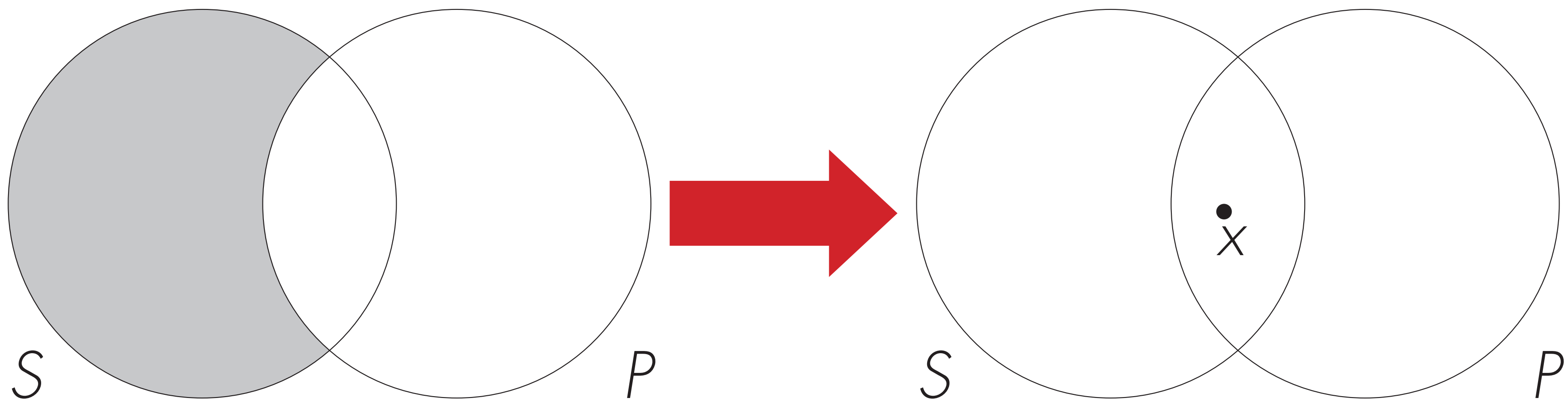
In general, any true **A** statement may be transformed into a true **I** statement about the same subject (*S*) and predicate (*P*) terms.*



*__Note:__ This also only works when the subject term is not empty!

Explanation

The idea is that there are no *S*'s outside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) in *P*.



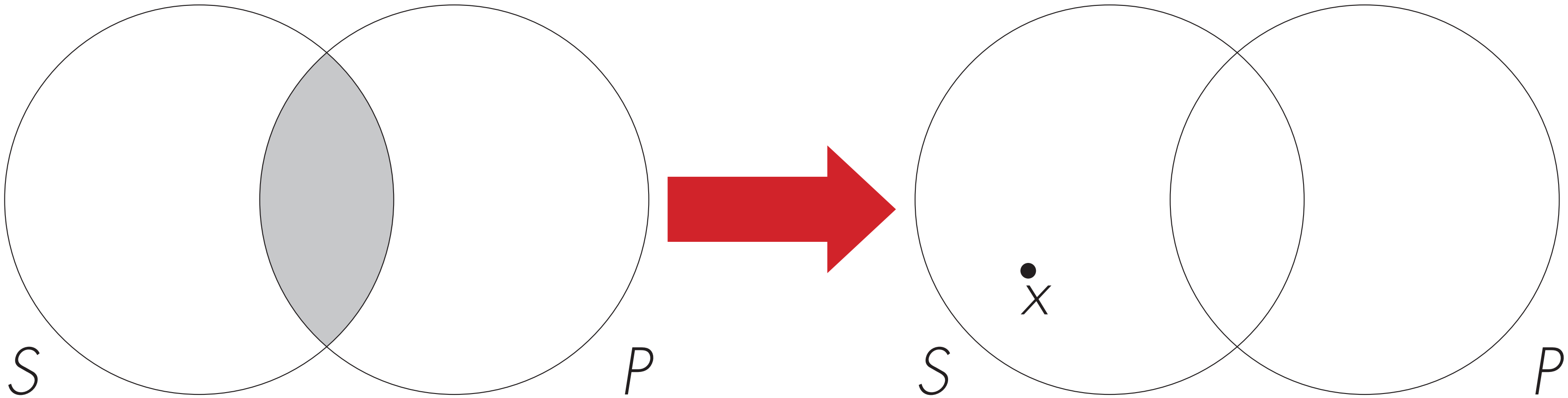
A Statement: All *S* is *P*.

I Statement: Some *S* is *P*.

***Note:** This also only works when the subject term is not empty!

Subalternation

Similarly, any true **E** statement may be transformed into a true **O** statement about the same subject (*S*) and predicate (*P*) terms.*



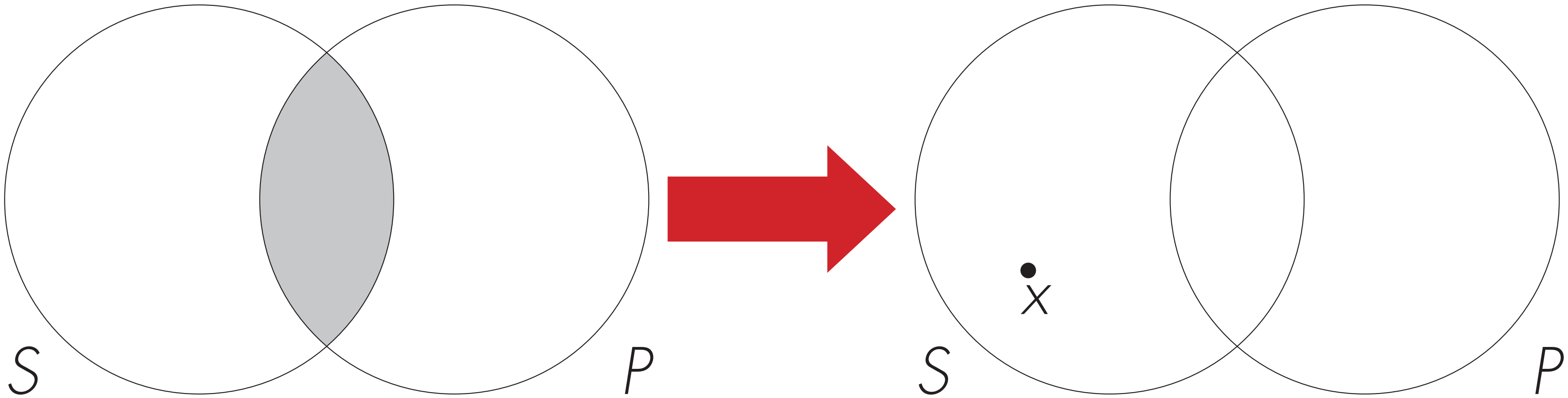
E Statement: No *S* is *P*.

O Statement: Some *S* is not *P*.

***Note:** This also only works when the subject term is not empty!

Explanation

The idea is that there are no *S*'s inside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) outside *P*.

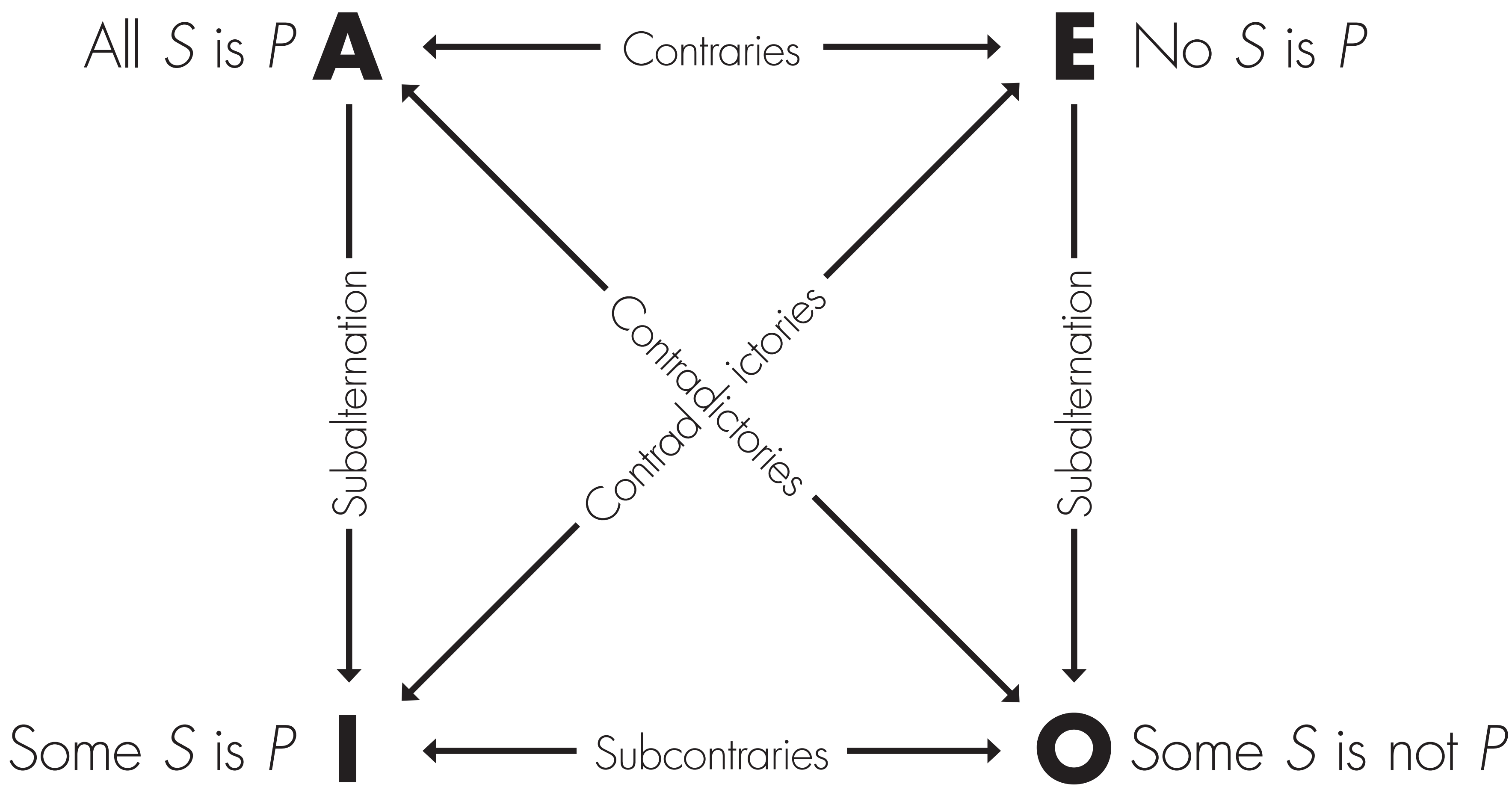


E Statement: No *S* is *P*.

O Statement: Some *S* is not *P*.

***Note:** This also only works when the subject term is not empty!

The Square of Opposition



Inferences from the Square

Fix the subject (*S*) and the predicate (*S*) terms. Then the square of oppositions reveals these inferences:

- If **A** is true: **E** is false; **I** is true; **O** is false.
- If **A** is false: **O** is true; **E** and **I** are undetermined.

- If **E** is true: **A** is false; **I** is false; **O** is true.
- If **E** is false: **I** is true; **A** and **O** are undetermined.

- If **I** is true: **E** is false; **A** and **O** are undetermined.
- If **I** is false: **A** is false; **E** is true; **O** is true.

- If **O** is true: **A** is false; **E** and **I** are undetermined.
- If **O** is false: **A** is true; **E** is false; **I** is true.

Inferences from the Square

The square of opposition contains a lot of useful information concerning what you can infer from a single categorical statement, but Venn diagrams provide more intuitive ways to figure out these inferences.

Next Class...

We will look at some further inferences that can be made from a single categorical statement.

Also, please do not forget to turn in your response to the Lecture #21 Questionnaire on your way out.