Introduction to Logical Reasoning

Lecture #19

Categorical Statements

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Categories

Category: A collection or set of things. A category is denoted by italicized capital letters, e.g., A, B, C, ...

E.g., P could be the category of professors teaching at CMU-Q.

Element: A thing that is in a category. An element is denoted by italicized lowercase letters, e.g., x, y, z, ...

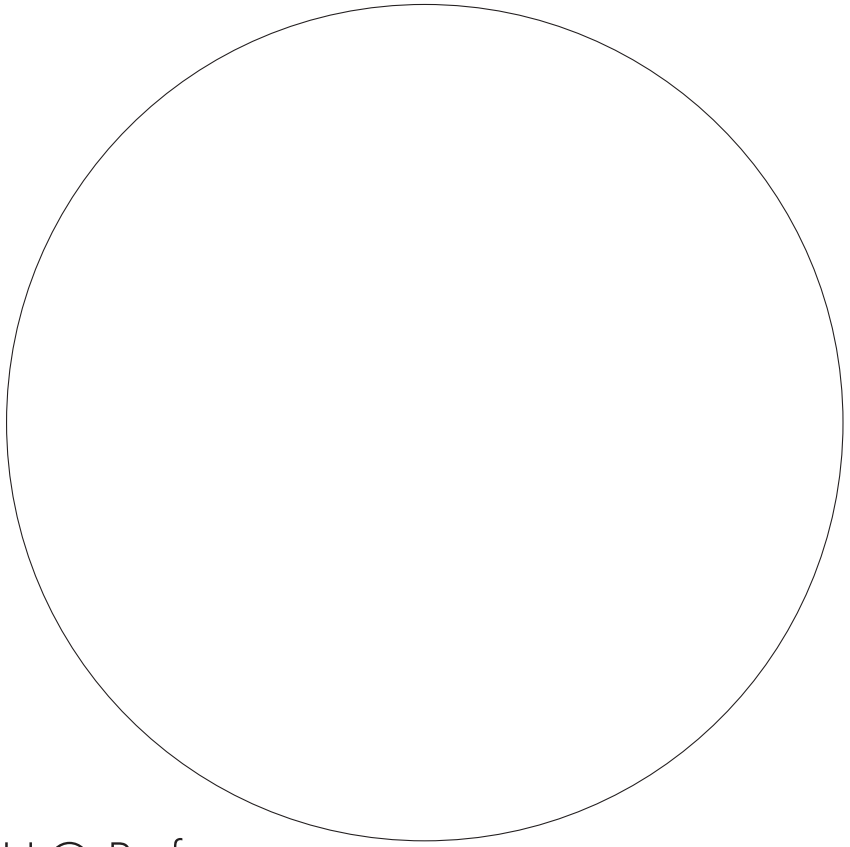
E.g., x could be Professor Gray, an element of category P.

Empty category: A category that contains no elements.

E.g., the category M of CMU-Q professors living on Mars is (currently) an empty category.

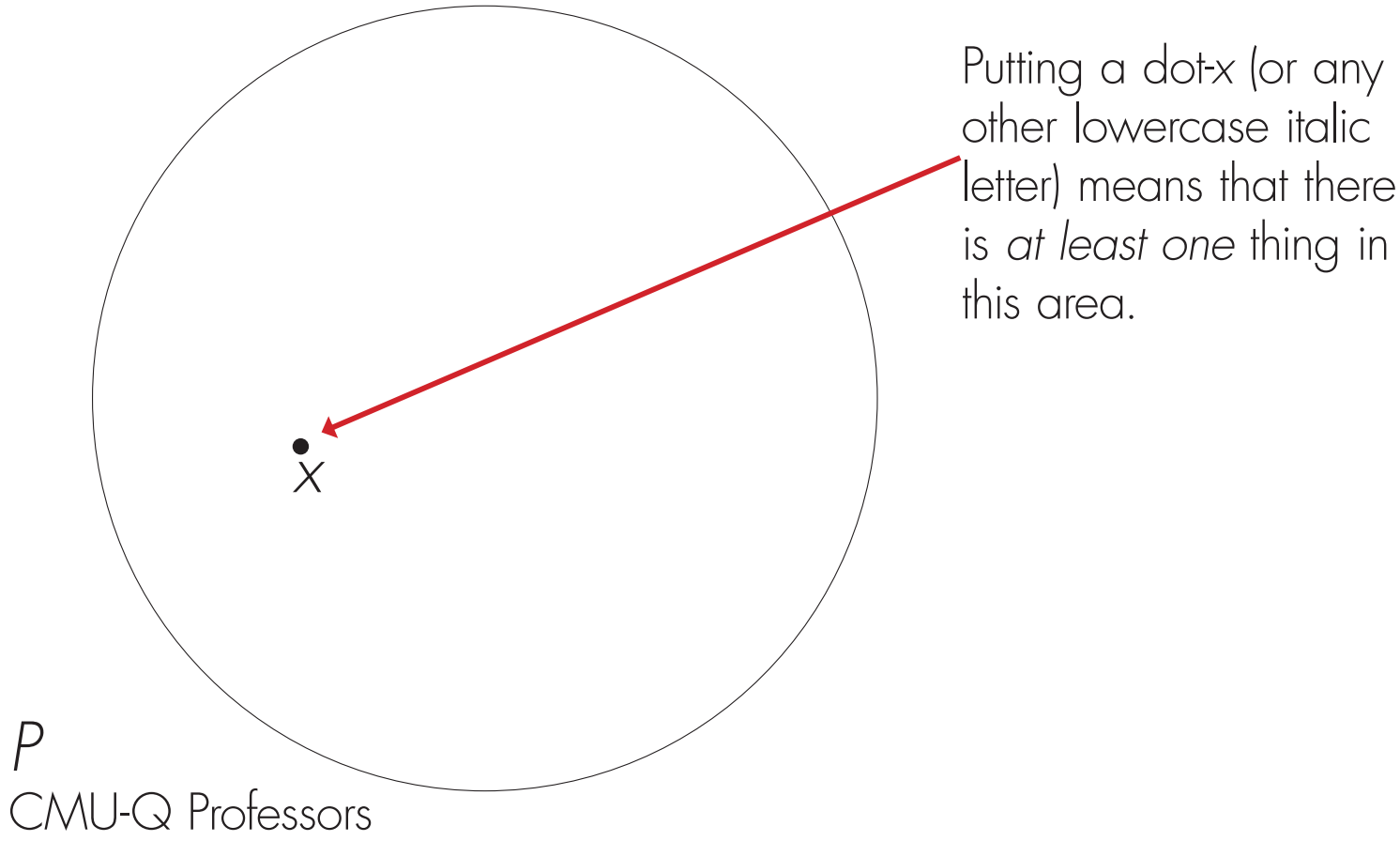
Venn Diagrams

Categories may be diagrammed using circles called Venn diagrams. For instance, category *P* (of CMU-Q professors) may be diagrammed:



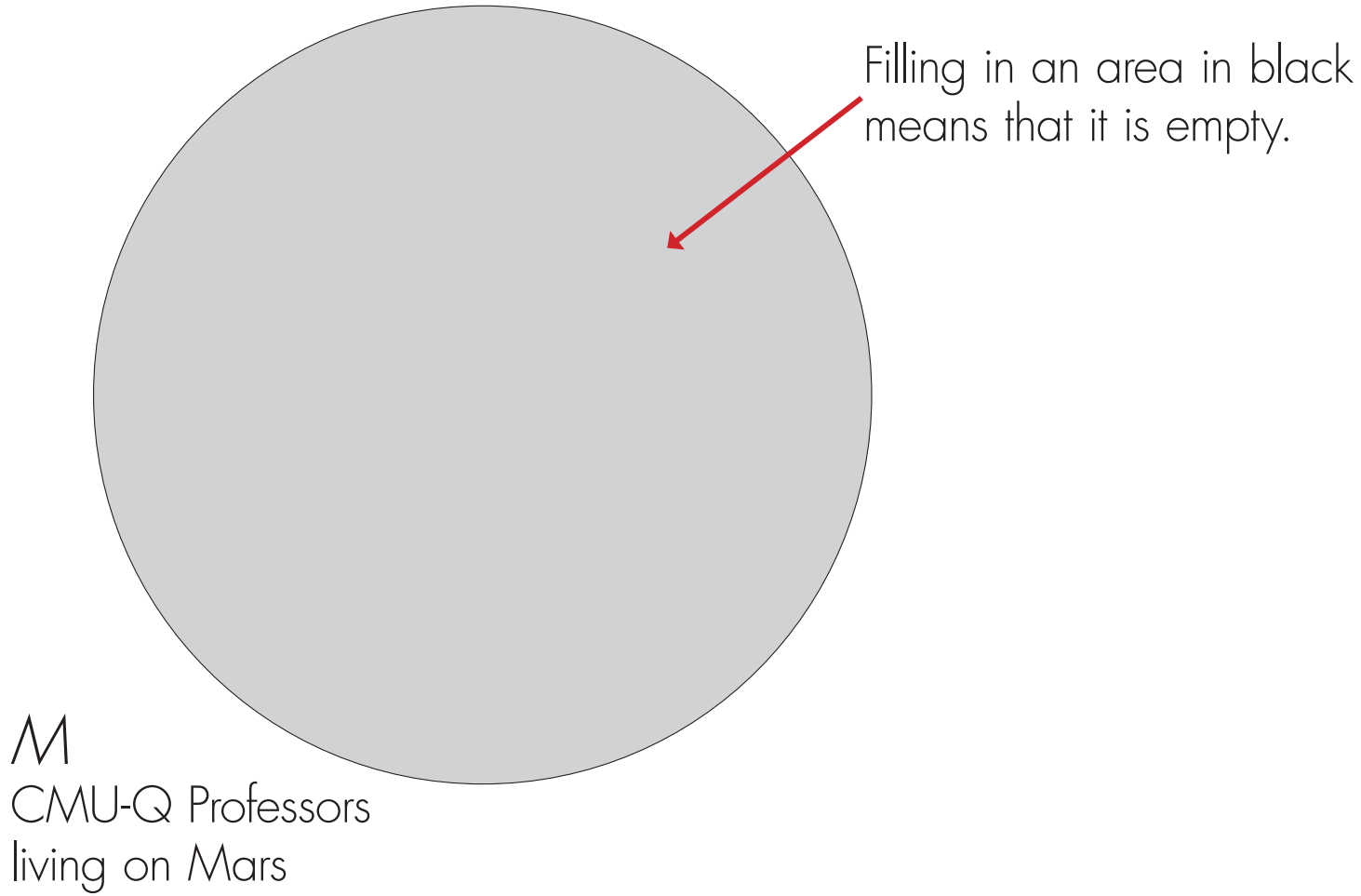
Venn Diagrams

This diagram can also illustrate that category *P* (CMU-Q professors) has at least one element *x* (Professor Gray):



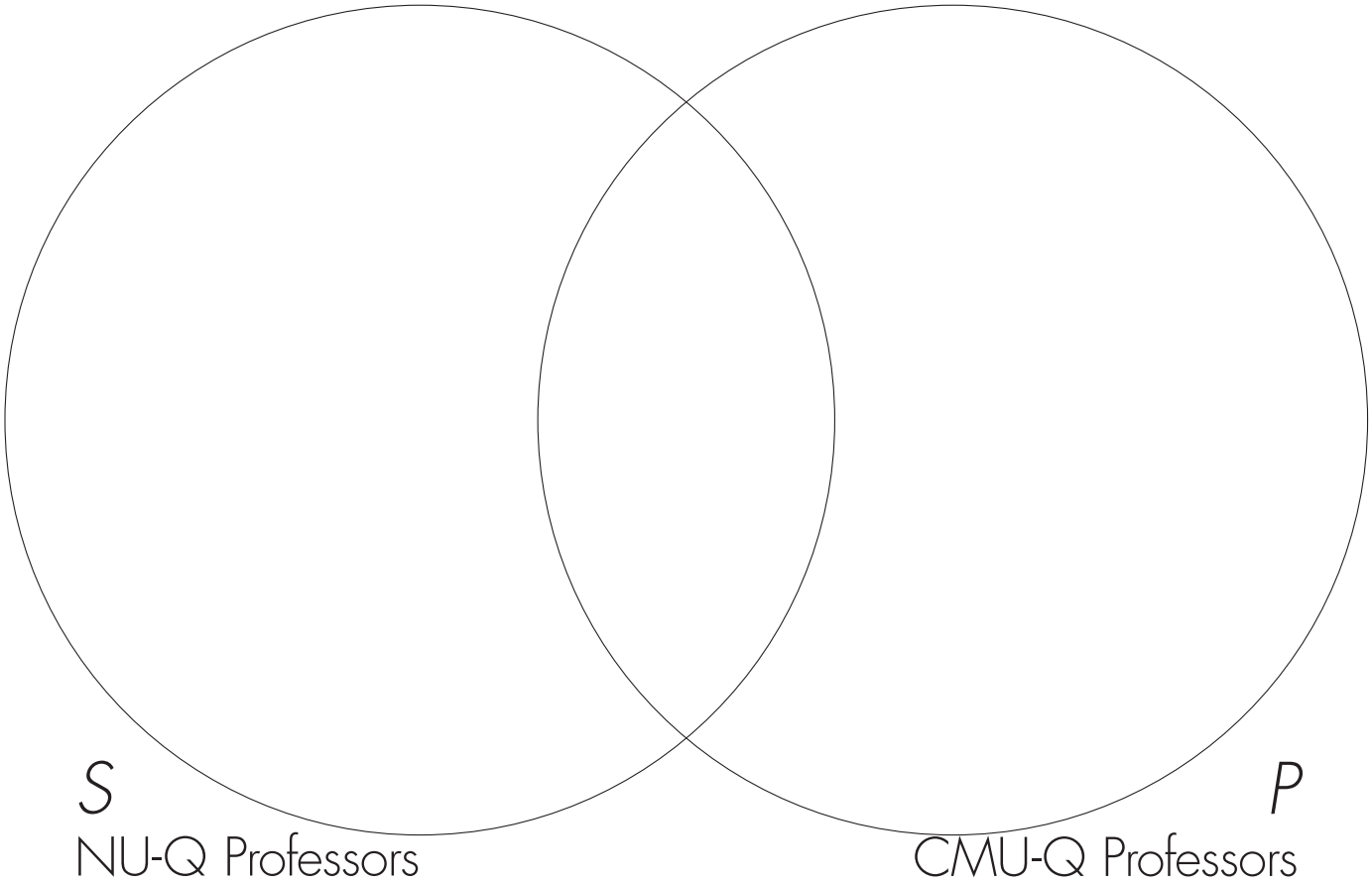
Venn Diagrams

A Venn diagram can also illustrate that category *M* (CMU-Q professors living on Mars) is empty:



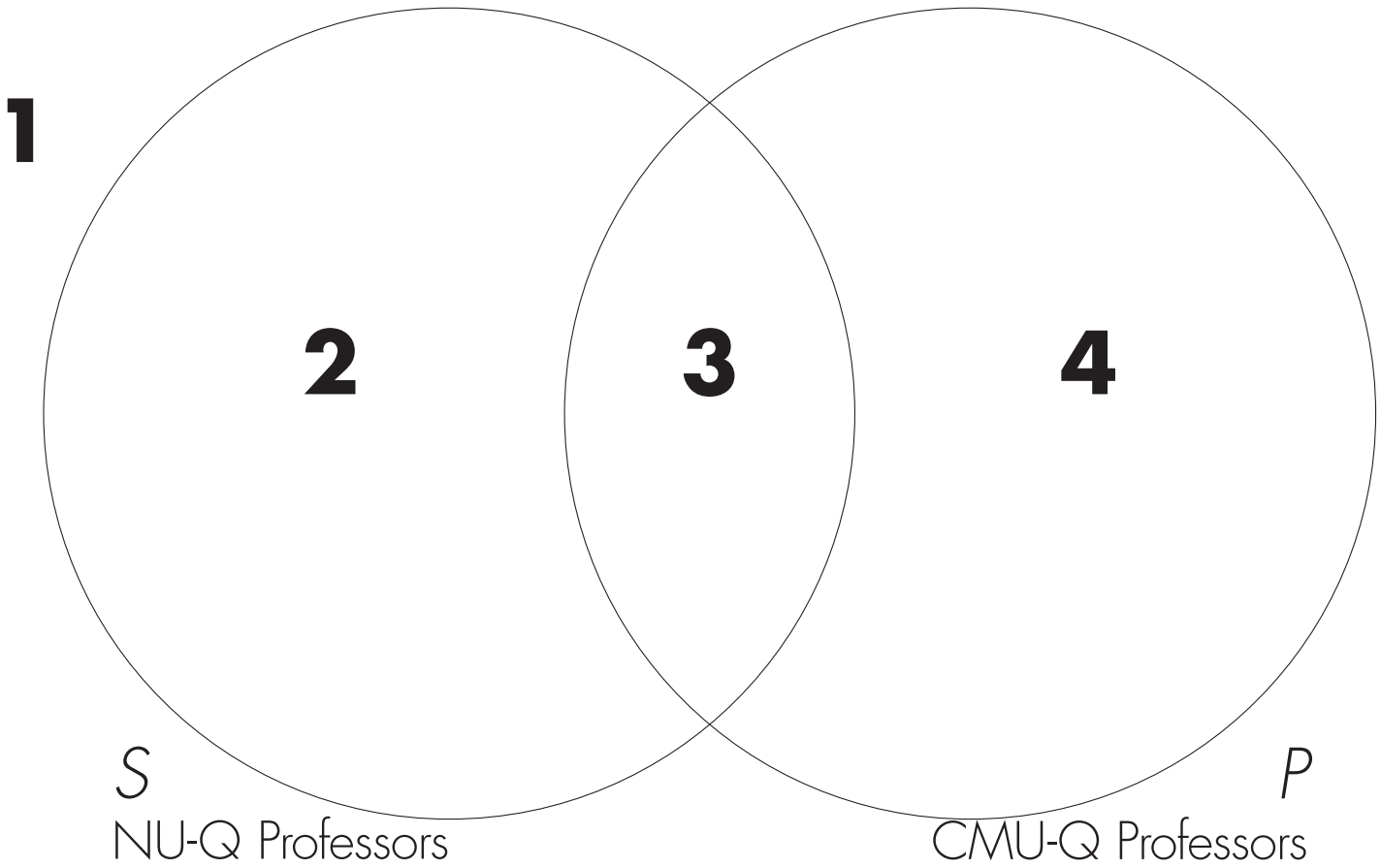
Venn Diagrams

A Venn diagram can also illustrate the possible relationships between two different categories. This is the diagram for categories *S* (NU-Q professors) and *P* (CMU-Q professors):



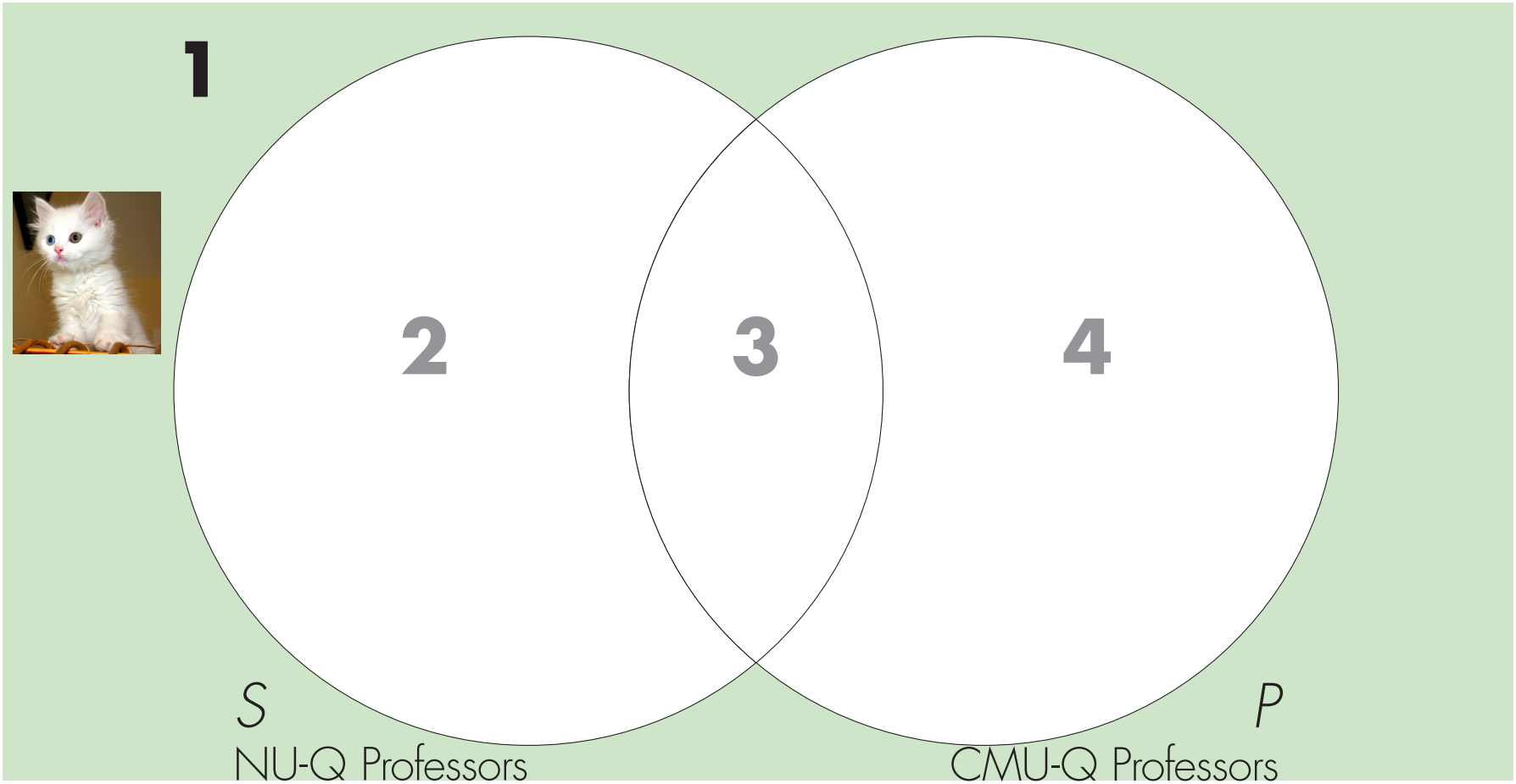
Venn Diagrams

Notice that the diagram reveals four **subcategories**, or "zones", involving categories *S* (NU-Q professors) and *P* (CMU-Q professors):



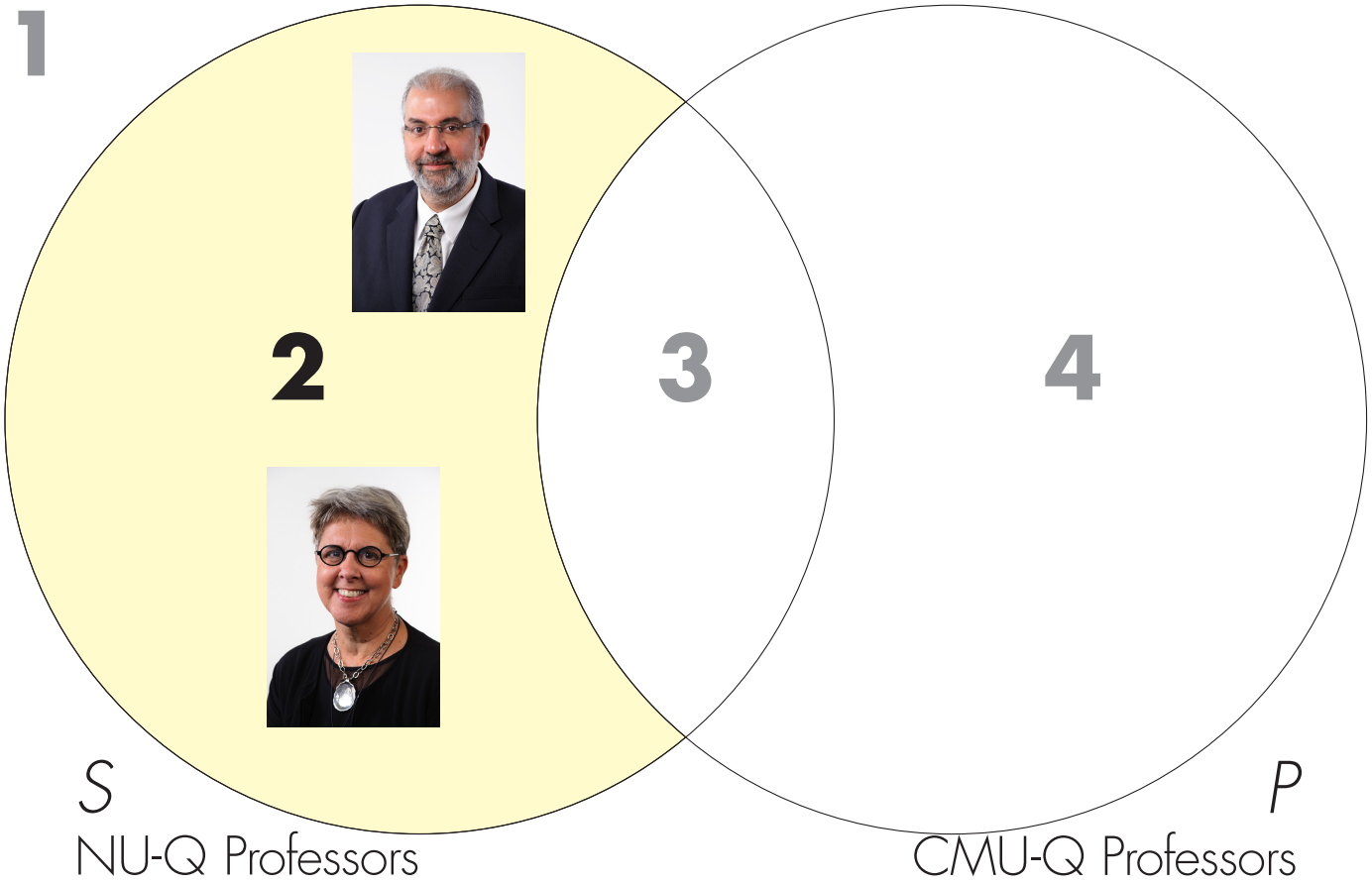
Venn Diagrams

Zone 1 ("donut") has things that are outside of *S* (NU-Q professors) and also outside of *P* (CMU-Q professors). It is made up of whatever is *neither* an *S nor* a *P*.



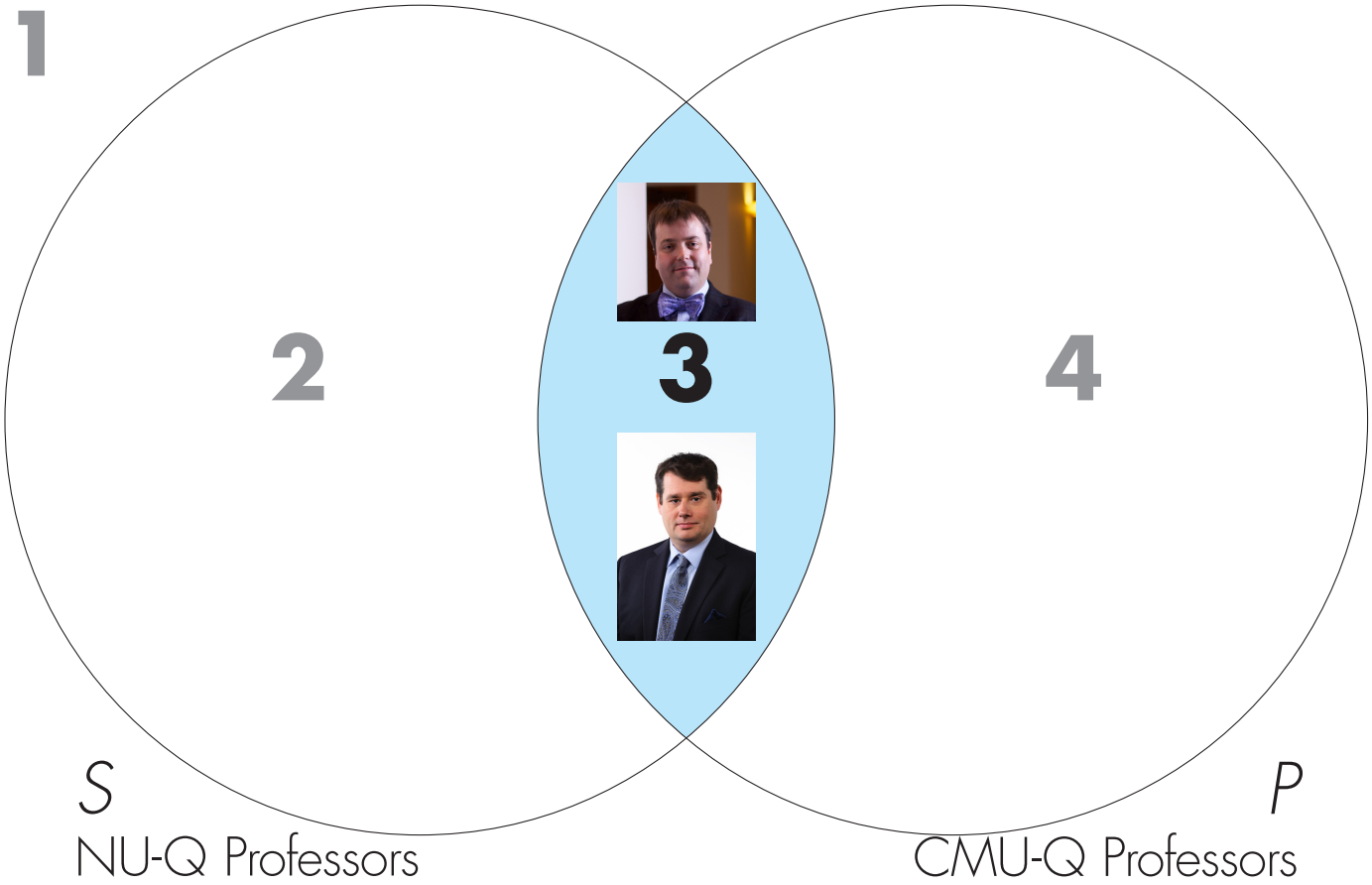
Venn Diagrams

Zone 2 ("Pacman") has things that are inside of *S* (NU-Q professors) but outside of *P* (CMU-Q professors). It is made up of whatever *is* an *S* but *not* a *P*.



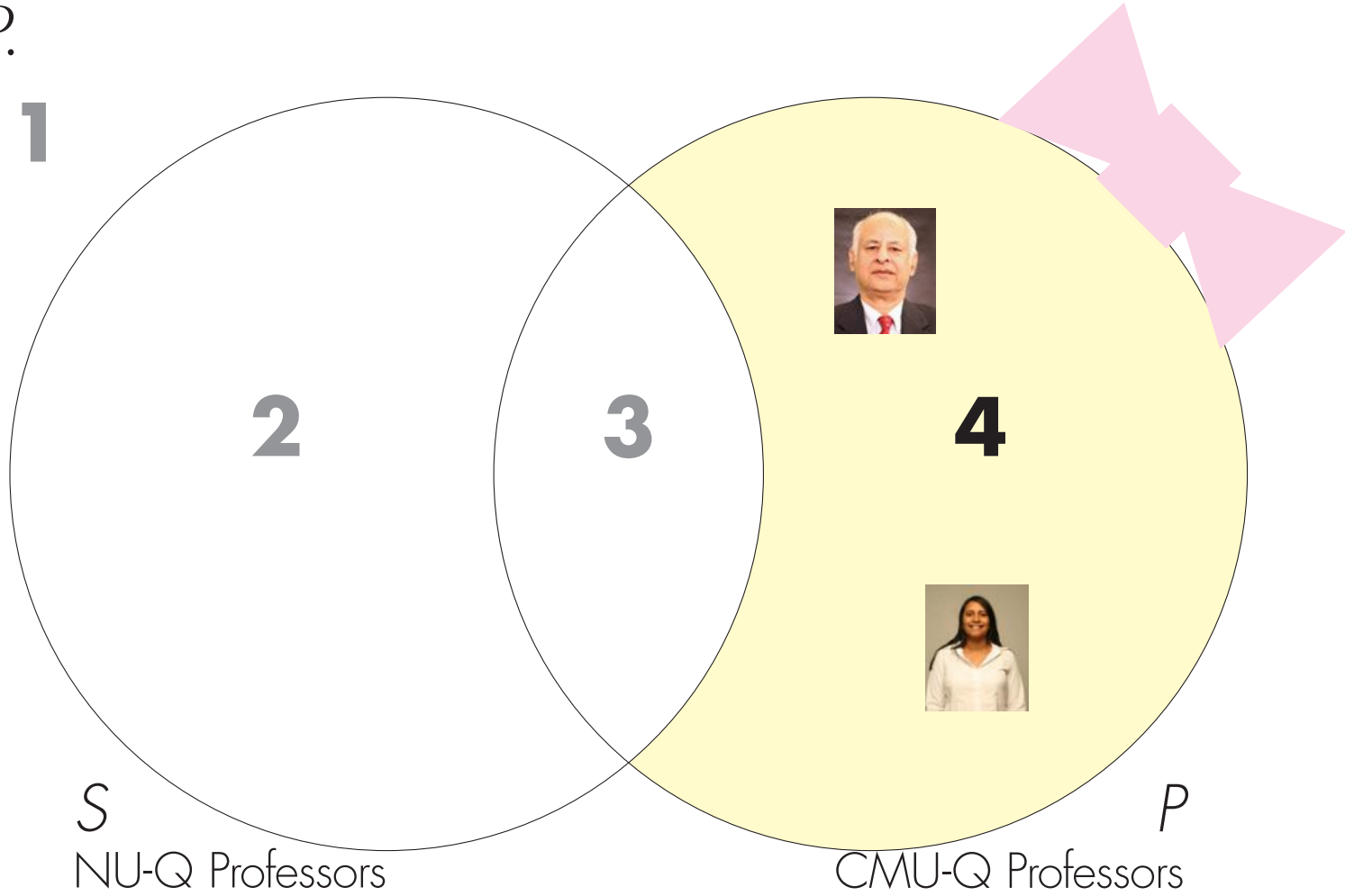
Venn Diagrams

Zone 3 ("teardrop") has things that are inside both *S* (NU-Q professors) and *P* (CMU-Q professors). It is made up of whatever is *both* an *S and* a *P*.



Venn Diagrams

Zone 4 ("Ms. Pacman") has things that are outside of *S* (NU-Q professors) but inside of *P* (CMU-Q professors). It is made up of whatever is *not* an *S* but *is* a *P*.



Categorical Statements

A **categorical statement** makes a claim concerning the relationship between two categories of things: the subject term (*S*) and the predicate term (*P*).

The **subject term** (*S*) names the main category the statement is about; the **predicate term** (*P*) names the category the statement is using to say something about that subject.

Categorical Statements

There are four standard forms of categorical statements:

1. Universal Affirmative (**A**): All *S* is *P*.

2. Universal Negative (**E**): No *S* is *P*.

3. Particular Affirmative (**I**): Some *S* is *P*.

4. Particular Negative(**O**): Some *S* is not *P*.

Categorical Statements

When analyzing a categorical statement, there are three questions to ask about it:

1. **Quantity:** Does it refer to *all* members of *S*, or only to *some* members of *S*?
2. **Quality:** Does the proposition *affirm* or *deny* some relationship between *S* and *P*?
3. **Distribution:** Does it refer to *all* members of *P* or only to *some* members of *P*?

Universal Affirmative (**A**)

All men are mortal.

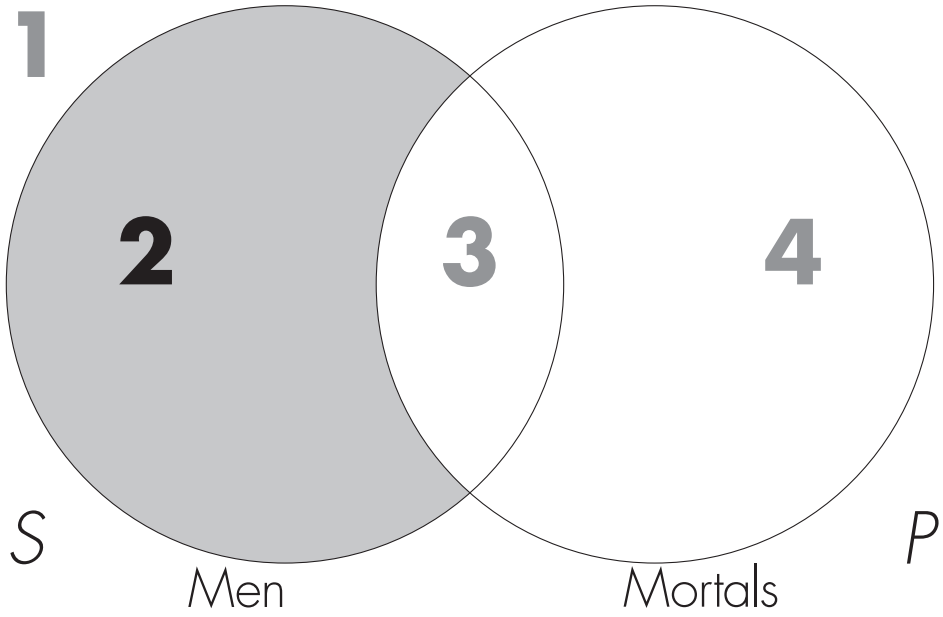
Subject (*S*): Men.

Predicate (*P*): Mortals.

Logical form: All *S* is *P*.

The idea is that the area of *S* that is *not* shared with *P* must be empty. All the *S*'s are inside *P*.

The Venn diagram:



For any statement of the form "All *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means all the *S*'s must be in that zone and all other *S* zones must therefore be empty. In this case, zone 2 is empty and so it is filled in.

I.e., **shade in all zones of *S not* shared with *P*.**

Universal Affirmative (**A**)

Quantity: Universal because it is referring to *all* the *S*'s. They are all *P*'s, as is seen in the Venn diagram.

Quality: Affirmative because it *affirms* that *S*'s are also *P*'s.

Distribution: *P* is *not* distributed because the statement only refers to *some* of the *P*'s, but not necessarily to all of them. As the Venn diagram shows, some of those *P*'s may be *S*'s, but some may not be.

Universal Negative (**E**)

No lawyers are honest.

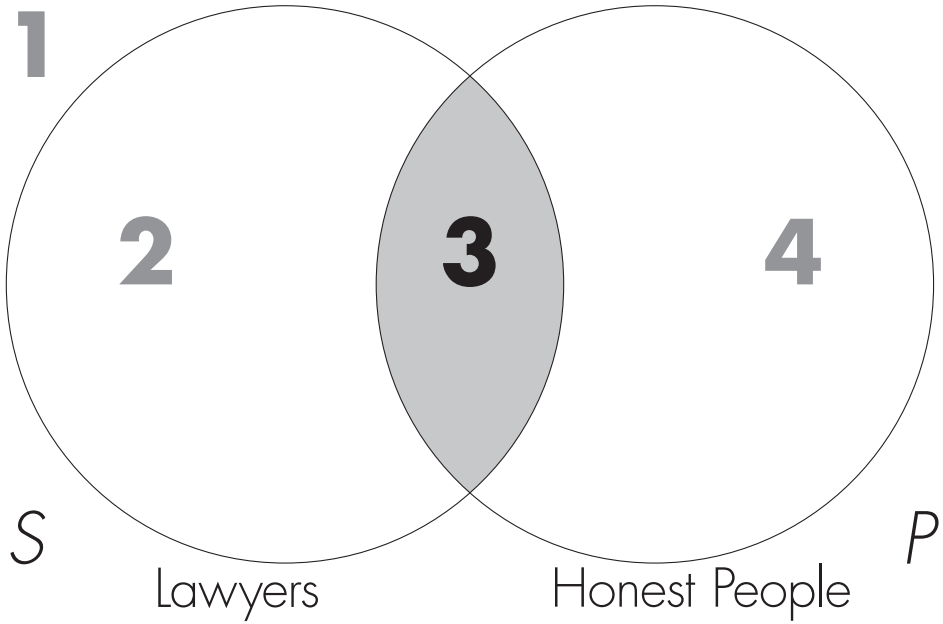
Subject (*S*): Lawyers.

Predicate (*P*): Honest people.

Logical form: No *S* is *P*.

The idea is that the area of *S* that *is* shared with *P* must be empty. None of the *S*'s are inside *P*.

The Venn diagram:



For any statement of the form "No *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means no *S*'s must be in that zone. In this case, zone 3 is empty and so it is filled in.

I.e., **shade in all zones of *S shared* with *P***.

Universal Negative (**E**)

Quantity: Universal because it is referring to *all* the *S*'s. They are all not *P*'s, as is seen in the Venn diagram.

Quality: Negative because it *denies* that *S*'s are also *P*'s.

Distribution: *P is* distributed because the statement refers to *all* of the *P*'s. As the Venn diagram shows, they are all not *S*'s.

Particular Affirmative (I)

Some students are hard workers.

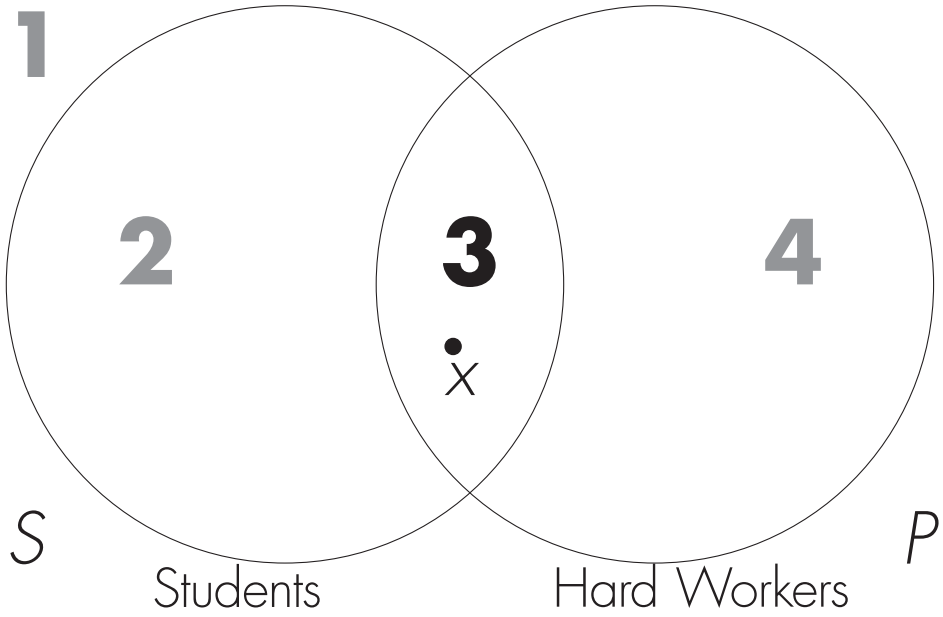
Subject (*S*): Students.

Predicate (*P*): Hard workers.

Logical form: Some *S* is *P*.

The idea is that the area of *S* that *is* shared with *P* must have something in it. At least one *S* is inside *P*.

The Venn diagram:



For any statement of the form "Some *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means there is at least one *S* in that zone. In this case, zone 3 has something and so it has a dot-*x* inside it.

I.e., **a dot-*x* goes in the zone of *S shared* with *P*.**

Particular Affirmative (I)

Quantity: Particular because it is referring to *some* of the *S*'s but not necessarily to all of them. Some *S*'s are *P*'s, but some may not be, as is seen in the Venn diagram.

Quality: Affirmative because it *affirms* that at least one *S* is also a *P*.

Distribution: *P* is *not* distributed because the statement only refers to *some* of the *P*'s, but not necessarily to all of them. As the Venn diagram shows, some *P*'s are *S*'s, but some may not be.

Particular Negative (O)

Some professors are not lazy.

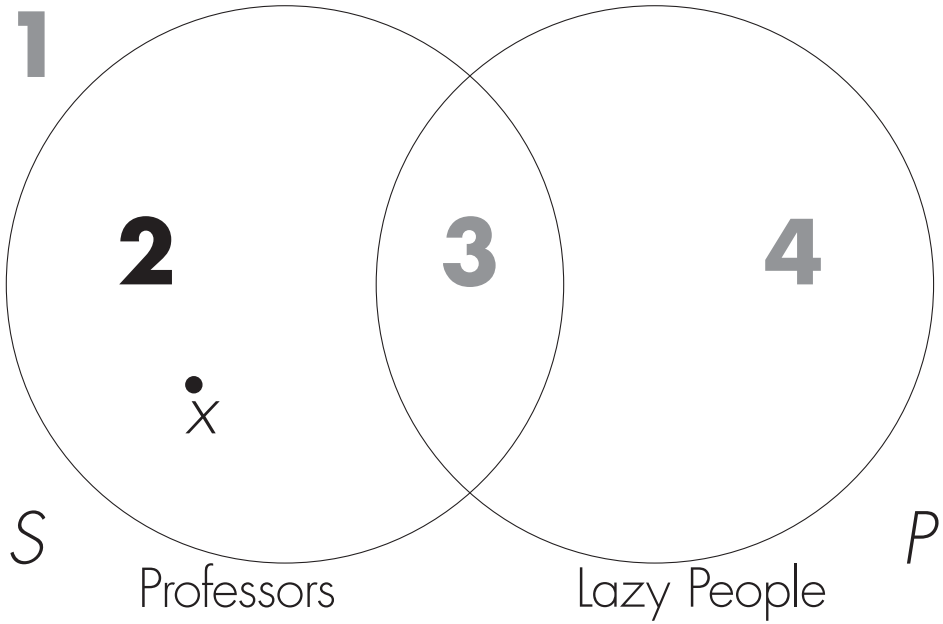
Subject (*S*): Professors.

Predicate (*P*): Lazy people.

Logical form: Some *S* is not *P*.

The idea is that the area of *S* that is *not* shared with *P* must have something in it. At least one *S* is not inside *P*.

The Venn diagram:



For any statement of the form "Some *S* is not *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means there is at least one *S* outside that zone. In this case, zone 2 has something and so it has a dot-*x* inside it.

I.e., **a dot-*x* goes in the zone of *S not* shared with *P*.**

Particular Negative (O)

Quantity: Particular because it is referring to some of the *S*'s but not necessarily to all of them. Some *S*'s are not *P*'s, but some may be, as is seen in the Venn diagram.

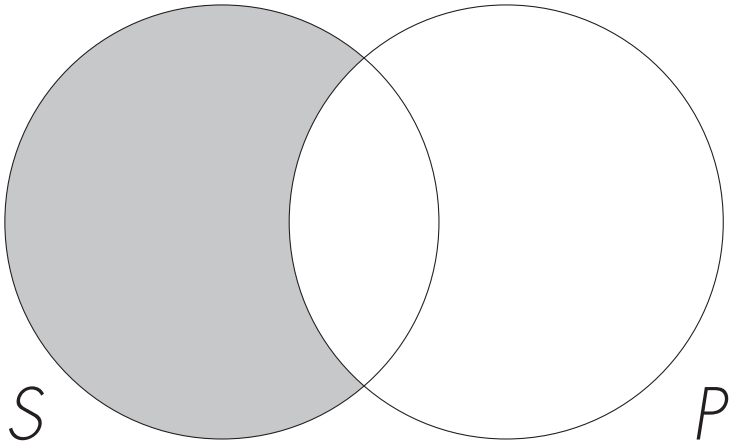
Quality: Negative because it *denies* that there are no *S*'s that are also not *P*'s.

Distribution: *P is* distributed because the statement refers to *all* of the *P*'s. As the Venn diagram shows, all *P*'s are not that *x* inside *S*.

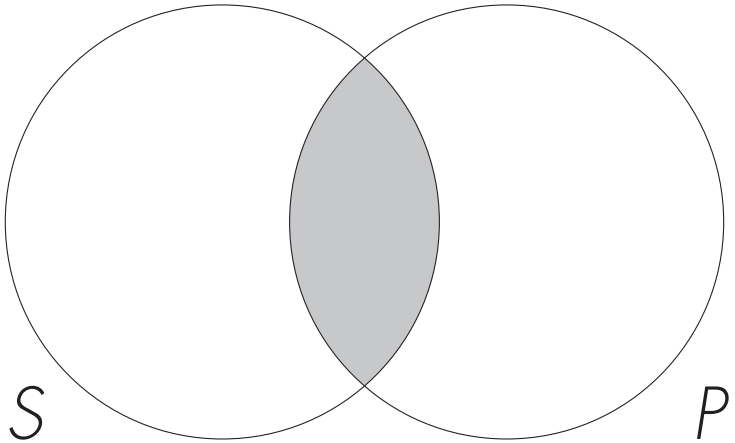
Categorical Statements

Just keep in mind, if you can draw a picture of the statement in a Venn diagram, then you can much more easily figure out its logical structure.

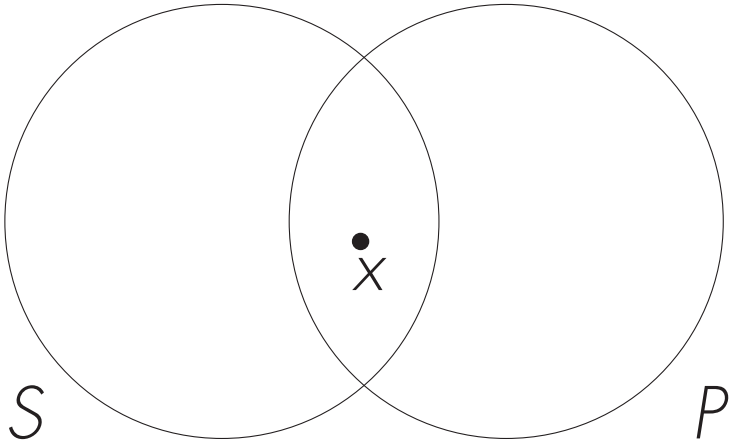
Four Standard Forms of Categorical Statements



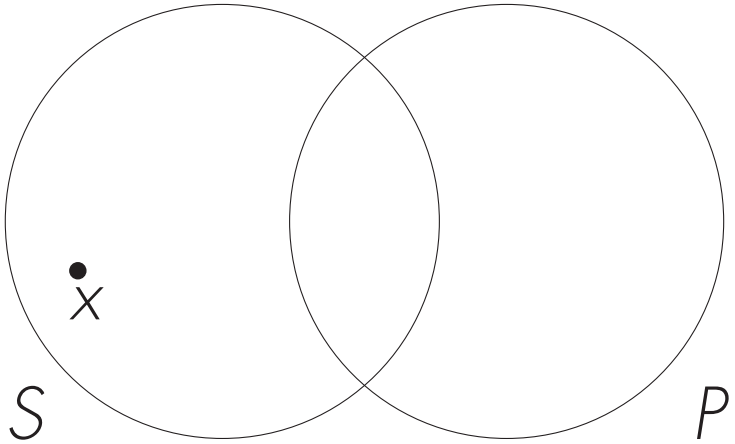
A: All *S* is *P*.
(Shade in all of *S* not shared with *P*.)



E: No *S* is *P*.
(Shade in all of *S* shared with *P*.)



I: Some *S* is *P*.
(Dot-*x* in *S* shared with *P*.)



O: Some *S* is not *P*.
(Dot-*x* in *S* not shared with *P*.)

Next Class...

We will examine more complicated forms of categorical propositions.

Also, please do not forget to turn in your response to the Lecture #19 Questionnaire on your way out.