Introduction to Logical Reasoning

Lecture #18

Advanced Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

1. $p \rightarrow q$.
2. p.

$\therefore$ q.

2. *Modus Tollens* (M.T.)

1. $p \rightarrow q$.
2. $\sim q$.

$\therefore$ $\sim p$.

3. Hypothetical Syllogism (H.S.)

1. $p \rightarrow q$.
2. $q \rightarrow r$.

$\therefore$ $p \rightarrow r$.

4. Disjunctive Syllogism (D.S.)

1. $p \vee q$.
2. $\sim p$.

$\therefore$ q.

5. Constructive Dilemma (C.D.)

1. $(p \rightarrow q)$ & $(r \rightarrow s)$.
2. $p \vee r$.

$\therefore$ $q \vee s$.

6. Absorption (Abs.)

1. $p \rightarrow q$.

$\therefore$ $p \rightarrow (p$ & $q)$.

7. Simplification (Simp.)

1. p & q.

$\therefore$ p.

8. Conjunction (Conj.)

1. p.
2. q.

$\therefore$ p & q.

9. Addition (Add.)

1. p.

$\therefore$ $p \vee q$.

Natural Deduction

Today we finally bring all of our skills in natural deduction together. We now look at proofs where we do not know in advance how many steps they will take to solve. However, the process remains the same.

Argument 1

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. $A \rightarrow B$.
2. $A \vee (C \,\&\, D)$.
3. $\sim B \,\&\, \sim E$.

$\therefore$ C.

Argument 2

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. $(\sim M \,\&\, \sim N) \rightarrow (O \rightarrow N)$.
2. $N \rightarrow M$.
3. $\sim M$.

$\therefore \sim O$.

Argument 3

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct this argument's formal proof of validity.

> **Layli** is present only if **Majnun** is happy. **Cala** is pleased if both **Layli** is present and **Majnun** is happy. **Dirran** being pleased is a necessary condition for both **Layli** being present and **Cala** being pleased. Therefore, **Layli** being present is sufficient for **Dirran** to be pleased. (L, M, C, D)

Learning Natural Deduction

There are only three ways to learn natural deduction:

1. Practice,
2. Practice, and
3. Practice.

If you do not practice this, then you will not be able to do it. I trust you now understand *modus ponens* and *modus tollens*, so you can follow the implications here.

Next Class...

We will have an in-class review session for **exam #2**.

The exam itself is this Thursday (November 7TH). It will cover the material from units #3, #4, and #5. It will be in **lecture hall 1202** and begin promptly at 3:30PM. Show up and be seated by that time.

In the meantime, keep practicing this material!

Otherwise, please do not forget to turn in your response to the Lecture #18 Questionnaire on your way out.