Introduction to Logical Reasoning

Lecture #17

Longer Proofs by Natural Deduction

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Natural Deduction

We have been constructing proofs where the steps are not given to us. To continue this process, today we now look at arguments whose proofs can be done in just *three* steps. Once you can do this, you will have the skills to tackle most proofs by natural deduction.

Argument 1

The following is a valid argument. Use natural deduction to construct that argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(A \vee B) \rightarrow \sim C$.
2. $C \vee D$.
3. A.

$\therefore$ D.

Argument 2

The following is a valid argument. Use natural deduction to construct that argument's formal proof of validity. This proof can be done in only *three* steps.

1. $(P \rightarrow Q) \,\&\, (Q \rightarrow P)$.
2. $R \rightarrow S$.
3. $P \vee R$.

$\therefore$ $Q \vee S$.

Natural Deduction

As I have said before, the real goal with natural deduction is to be able to take arguments in English, translate them into the language of logic, and then formally prove their validity.

Argument 3

The following is a valid argument in English. (1) Translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct that argument's formal proof of validity. This proof can be done in only *three* steps.

> **Majnun** being happy is necessary for **Layli** being present. **Majnun** being happy is sufficient for either **Cala** or **Dirran** being pleased. **Cala** is not pleased. **Layli** is present. Therefore, **Dirran** is pleased. (L, M, C, D)

Learning Natural Deduction

There are only three ways to learn natural deduction:

1. Practice,
2. Practice, and
3. Practice.

If you do not practice this, then you will not be able to do it. I trust you now understand *modus ponens* and *modus tollens*, so you can follow the implications here.

Next Class...

We will do a workshop on creating formal proofs of validity that can be done in either two or three steps.

Also, please do not forget to turn in your response to the Lecture #17 Questionnaire on your way out.