Introduction to Logical Reasoning

Lecture #13

Argument Patterns

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Modus Ponens (M.P.)

Consider the following argument:

> **Studying** hard is a sufficient condition for **passing** the class. I **study** hard. Therefore, I **pass** the class.

This can be formalized in standard argumentative form as follows:

1. $S \rightarrow P$.
2. S.

$\therefore$ P.

Modus Ponens (M.P.)

A truth table shows that this argument is *valid*:

Premise 2 S	Conclusion P	Premise 1 S → G
T	T	T
T	F	F
F	T	T
F	F	T

In all instances where the premises are all true, so is the conclusion. So it is logically impossible to have true premises but a false conclusion.

Modus Ponens (M.P.)

This argument has the following general pattern, which is known as ***modus ponens*** (M.P.):

1. $p \rightarrow q$.
2. p.

$\therefore$ q.

So any inference that has this form, by affirming (1) a hypothetical $p \rightarrow q$ and (2) its antecedent p to imply ($\therefore$) affirming its consequent q, is always logically valid.

Note: The use of lower-case, italic letters p and q, means that *any* two generic statements can be combined into the argument pattern of M.P.

Identifying Patterns

This same pattern of M.P. may appear in arguments that appear to be way more complicated:

> If I **study** hard and I **attend** every class, then I either **pass** the class or **die** trying. I **study** hard and I **attend** every class. Therefore, I either **pass** the class or **die** trying.

Notice this is a (1) a (complicated) hypothetical and (2) its (conjunctive) antecedent implying (∴) affirming its (disjunctive) consequent.

Identifying Patterns

This can be seen more clearly when formalizing the argument:

1. $(S \& A) \to (P \lor D)$.
2. $S \& A$.

$\therefore\ P \lor D$.

You can then see the pattern of M.P. start to emerge:

1. $(\overbrace{S \& A}^{p}) \to (\overbrace{P \lor D}^{q})$.
2. $\overbrace{S \& A}^{p}$.

$\therefore\ \overbrace{P \lor D}^{q}$.

Argument Patterns

Knowing commonly used argument patterns is extremely useful. Once you know that a particular pattern is logically valid, if you see that same pattern appear in another argument, you then know right away that this new argument is also logically valid.

So, for instance, any argument that has M.P.'s pattern—no matter what content statements *p* and *q* may have, and no matter whether they positive, negative, or compound—is logically valid.

Argument 1

Consider the following argument:

> You can vote if you are eighteen. You are eighteen. Therefore you can vote.

Does this argument have the pattern of M.P.?

Argument 2

Consider the following argument:

> If you are eighteen, then you can vote. You are not eighteen. Therefore you cannot vote.

Does this argument have the pattern of M.P.?

Modus Tollens (M.T.)

Another common argument pattern is known as ***modus tollens*** (M.T.):

1. $p \rightarrow q$.
2. $\sim q$.

$\therefore$ $\sim p$.

In this case (1) affirming a hypothetical statement but (2) denying its consequent is meant to imply (∴) denying its antecedent.

Modus Tollens (M.T.)

And a truth table shows that this pattern is also valid:

		Premise 1	Premise 2	Conclusion
p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

In all instances where the premises are all true, so is the conclusion. So it is logically impossible to have true premises but a false conclusion.

Identifying Patterns

So anytime you see an inference that (1) affirms a hypothetical but (2) denies its consequent to conclude by (∴) denying its antecedent, this is logically valid. This still holds when these things get more complex:

1. $(A \rightarrow B) \rightarrow {\sim}(C \vee D).$

2. ${\sim}{\sim}(C \vee D).$

$\therefore\ {\sim}(A \rightarrow B).$

1. $\overbrace{(A \rightarrow B)}^{p} \rightarrow \overbrace{{\sim}(C \vee D)}^{q}.$

2. ${\sim}\overbrace{{\sim}(C \vee D)}^{q}.$

$\therefore\ {\sim}\overbrace{(A \rightarrow B)}^{p}.$

Argument 3

Consider the following argument:

> If you are eighteen, then you can vote. You cannot vote. Therefore you are not eighteen.

Does this conform to either argument pattern of M.P. or M.T.?

Fallacy of Affirming the Consequent

Now all argument patterns are good, however. Consider the following common argument pattern:

1. $p \rightarrow q$.
2. q.

$\therefore$ p.

The pattern here is affirming both (1) a hypothetical and (2) its consequent in order to conclude (∴) by affirming its antecedent.

Fallacy of Affirming the Consequent

A truth table shows that this pattern is invalid:

Conclusion	Premise 2	Premise 1
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

In line 3, both premises are true while the conclusion is false. So it is logically possible to have true premises and a false conclusion.

Fallacy of Affirming the Consequent

This is an extremely common fallacy known as the **fallacy of affirming the consequent**. For instance:

> If I have good business skills, then I will earn a lot of money. I earn a lot of money. Therefore, I have good business skills.

On a quick read this (rather common) argument may seem logically valid. But on closer inspection, it has the same pattern as this fallacy. So it is invalid!

Fallacy of Denying the Antecedent

Here is another bad argument pattern:

1. $p \rightarrow q$.
2. $\sim p$.

$\therefore$ $\sim q$.

The pattern here is (1) affirming a hypothetical but (2) denying its antecedent in order to conclude (∴) by denying its consequent.

Fallacy of Denying the Antecedent

And a truth table shows that this pattern is also invalid:

		Premise 1	Premise 2	Conclusion
p	q	$p \rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

In line 3, both premises are true while the conclusion is false. So it is logically possible to have true premises and a false conclusion.

Fallacy of Denying the Antecedent

This is another extremely common fallacy known as the **fallacy of denying the antecedent**. For instance:

> If science can prove that God is dead, then God is dead. But science cannot prove that God is dead. Therefore, God is not dead.

On a quick read this may seem logically valid. But on closer inspection, it has the same pattern as this fallacy. So it is invalid!

Valid vs. Invalid Patterns

It is sometimes easy to confuse a valid argument with a fallacy, so you need to be on guard!

- Do not confuse M.P. (affirming the *antecedent*) with the fallacy of affirming the *consequent*, and
- Do not confuse M.T. (denying the *consequent*) with the fallacy of denying the *antecedent*.

Next Class...

We will do a workshop on using truth tables to assess the validity of arguments.

We will work more on identifying argument patterns in the next unit on natural deduction.

Otherwise, please do not forget to turn in your response to the Lecture #12 Questionnaire on your way out.