Introduction to Logical Reasoning

Lecture #12

Validity & Truth Tables

Professor David Emmanuel Gray

Carnegie Mellon University in Qatar
Northwestern University in Qatar

Arguments

Recall that an **argument** is a collection of statements about which the claim is made that the truth of all the premises entails the truth of the conclusion.

So an argument asserts that the conclusion can be inferred from the premises. That is, the claim is that *if* the premises, then the conclusion *must* be true as well.

Valid and Invalid Arguments

Also recall the distinction between deductively valid and deductively invalid arguments:

A **valid** argument is an argument where the truth of all its premises logically entails the truth of its conclusion. It is logically impossible for the conclusion to be false while the premises are all true.

An **invalid** argument is an argument where it *is* logically possible for the conclusion to be false while the premises are all true.

Argument 1

Consider the following argument:

> If science can prove that God is dead, then God is dead. But science cannot prove that God is dead. Therefore, God is not dead.

Is this a logically valid argument? That is, does the conclusion logically follow from the premises?

Assessing Validity

Step 1: Put the argument into argumentative form:

1. If science can prove that God is dead, then God is dead.
2. Science cannot prove that God is dead.
∴ God is not dead.

Assessing Validity

Step 2: Label each simple positive statement in the argument.

Argument 1 involves two simple positive statements:

      S: Science can prove that God is dead, and

      G: God is dead.

Assessing Validity

Step 3: Translate the argument into the language of symbolic logic.

Here is argument 1:

1. $S \rightarrow G$.
2. $\sim S$.
   —————————
   $\therefore \sim G$.

Assessing Validity

Step 4: Construct a truth table.

The premises and conclusion must each have its own column.

Assessing Validity

Begin by putting each premise and the conclusion at the top of a column in the table, marking each of these (as premise or conclusion) for later reference:

Premise 1	Premise 2	Conclusion
$S \rightarrow G$	~S	~G

Put negative and compound statements (i.e., more than one letter/symbol, as in ~S, ~G, and $S \rightarrow G$ in argument 1) to the right. Put simple positive statements (i.e., only one letter, though none so far in argument 1) to the far left.

Assessing Validity

Add any additional columns, following the procedure for truth table construction:

G	S	Premise 1	Premise 2	Conclusion
		S → G	~S	~G

For argument 1, we have a ~G column, we need a G column. The ~S column requires an S column. With those two new G and S columns added, the S → G column is already broken down as well.

Assessing Validity

Fill in the rows, following the procedure for truth table construction:

G	S	Premise 1 S → G	Premise 2 ~S	Conclusion ~G
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Since there are only 2 simple positive statements in the table, it has $2^2 = 4$ rows.

Assessing Validity

Step 5: Circle any rows in which *all the premises* are true:

G	S	Premise 1 $S \rightarrow G$	Premise 2 $\sim S$	Conclusion $\sim G$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

For argument 1, the premises are both true in lines 2 and 4, so those are the rows that are circles. Row 1 is not circled because premise 2 is false in it; row 3 is not circled because both premises are false in it.

Assessing Validity

Step 6: Now circle the conclusion in these rows:

G	S	Premise 1 $S \rightarrow G$	Premise 2 $\sim S$	Conclusion $\sim G$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Assessing Validity

An argument is *valid* just when the conclusion is true in *all* the circled rows: that means it is logically impossible for true premises but a false conclusion.

G	S	Premise 1 S → G	Premise 2 ~S	Conclusion ~G
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

For argument 1, however, the conclusion is false in row 2. So argument 1 is *invalid*: line 2 shows that it *is* logically possible for true premises but a false conclusion.

Argument 1

Consider the following argument:

> If science can prove that God is dead, then God is dead. But science cannot prove that God is dead. Therefore, God is not dead.

Is this a logically valid argument? That is, does the conclusion logically follow from the premises?

> No, this is an invalid argument. Even if the premises are true, it is still logically possible that the conclusion is false.

Argument 2

Consider the following argument:

> God being dead is a necessary condition for science proving that God is dead. But God is not dead. Therefore, science cannot prove that God is dead.

Is this a logically valid argument? That is, does the conclusion logically follow from the premises?

Assessing Validity

Step 1: Put the argument into argumentative form:

1. God being dead is a necessary condition for science proving that God is dead.
2. God is not dead.

∴ Science cannot prove that God is dead.

Assessing Validity

Step 2: Label each simple positive statement in the argument.

Argument 2 involves two simple positive statements:

> S: Science can prove that God is dead, and

> G: God is dead.

Assessing Validity

Step 3: Translate the argument into the language of symbolic logic.

Here is argument 2:

1. $S \rightarrow G$.
2. $\sim G$.
$$\therefore \sim S.$$

Assessing Validity

Step 4: Construct a truth table.

G	S	Premise 1 S → G	Premise 2 ~G	Conclusion ~S
T	T	T	F	F
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Assessing Validity

Step 5: Circle any rows in which all the premises are true.

G	S	Premise 1 $S \rightarrow G$	Premise 2 ~G	Conclusion ~S
T	T	T	F	F
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Assessing Validity

Step 6: Now circle the conclusion in that row.

G	S	Premise 1 $S \rightarrow G$	Premise 2 ~G	Conclusion ~S
T	T	T	F	F
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Assessing Validity

An argument is *valid* just when the conclusion is true in all the circled rows: that means it is logically impossible for true premises but a false conclusion.

		Premise 1	Premise 2	Conclusion
G	S	S → G	~G	~S
T	T	T	F	F
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

For argument 2, the conclusion is true in the circled row. So it is *valid*: whenever the premises are true, so is the conclusion.

Argument 2

Consider the following argument:

>God being dead is a necessary condition for science proving that God is dead. But God is not dead. Therefore, science cannot prove that God is dead.

Is this a logically valid argument? That is, does the conclusion logically follow from the premises?

>Yes, this is a valid argument. Whenever the premises are true, it is logically impossible for the conclusion to be false.

Next Class...

We will practice using truth tables to assess some common valid and invalid patterns that deductive arguments often take.

Also, please do not forget to turn in your response to the Lecture #12 Questionnaire on your way out.