Introduction to Logical Reasoning

Lecture #10

Symbolic Logic & Natural Language

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The "Alphabet" of Symbolic Logic

***Specific* positive simple statements** are represented by upper-case, upright letters A, B, C, D, . . . , Z.

***Generic* statements** (that is, statements that could be anything: positive, negative, compound, or any combination thereof) are represented by lower-case, italic letters $p, q, r, \ldots, z$.

Four **logical operators/connectives** are represented by &, ~, ∨, →.

Grouping punctuation is represented by (,), [,], {, }.

Conjunction

Recall that a **conjunctive statement** asserts the truth of *all* its statements. It is symbolized using & (called "ampersand").

So the conjunctive statement p & q asserts that statements p and q are both true. In this example, p and q are the **conjuncts**.

Note: By using the lower-case, italic letters p and q, this means that *any* two generic statements can be connected together as the conjuncts within a conjunctive statement.

Conjunction: Basic Example

Consider the following conjunctive statement:

Logic is fun **and** logic is hard.

The conjuncts are simple positive statements, which are symbolized:

F: Logic is **fun**.

H: Logic is **hard**.

The entire conjunctive statement is then symbolized as F & H.

Note: We are *now* using the upper-case, upright letters F and H because each represents a specific, simple positive statement.

Conjunction: Further Examples

As we have seen before, there are a lot of *other* ways to express the *exact same logic* symbolized by F & H:

Logic is fun **and** hard.

Logic is **both** fun **and** hard.

Logic is fun, **also** it is hard.

Logic is fun **but** hard.

Logic is fun, **yet** it is hard.

Logic is fun, **though** it is hard.

These certainly have different *connotations*, but they all have the same *logical* content.

Negation

Recall that a **negative statement** asserts that a given statement is false. It is symbolized using ~ (called "tilde").

So the negative statement ~p asserts that statement p is false.

Note: Once more, the use of a lower-case, italic letter, p, means that now *any* generic statement can be negated, whereas before we only considered the negative of simple statements.

Negation: Basic Example

Consider the following negative statement:

Logic is **not** an easy class.

This is made up of one simple positive statement, which is symbolized:

E: Logic is an **easy** class.

So the entire negated statement is symbolized as ~E.

Note: Once again, we use an upper-case, upright letter, E, to represent a specific, simple positive statement.

Negation: Further Examples

Not surprisingly, there are a lot of *other* ways to express the *exact same logic* symbolized by ~E:

> **It is false** that logic is an easy class.
>
> **It is not the case** that logic is an easy class.
>
> **It is not true** that logic is an easy class.

Disjunction

Recall that a **disjunctive statement** asserts the truth of *at least one* of its statements. It is symbolized using ∨ (called "wedge").

So the disjunctive statement $p \vee q$ asserts that at least one of statements p and q is true. In this example, p and q are the **disjuncts**.

Note: Yet again, the use of the lower-case, italic letters p and q means that *any* two generic statements can be connected together as the disjuncts within a disjunctive statement.

Disjunction: Basic Example

Consider the following disjunctive statement:

Logic is fun **or** logic is hard.

The disjuncts are simple positive statements, which are symbolized:

F: Logic is **fun**.

H: Logic is **hard**.

So the entire statement is symbolized as $\mathrm{F} \vee \mathrm{H}$.

Note: Yup, we using those upper-case, upright letters F and H to represent the specific, simple positive statements involved.

Disjunction: Further Examples

As we have also already seen before, there are a lot of *other* ways to express the *exact same logic* symbolized by $F \vee H$:

Logic is fun **or** hard.

Logic is **either** fun **or** hard.

Logic is fun **unless** it is hard.

As before, these may have different connotations, but they are all logically identical.

Inclusive vs. Exclusive

The word 'or' (and especially the word 'unless') can be used in two slightly different, but significant, ways.

Logic is fun **or** hard.

This is probably best described as ***inclusive*** **disjunction**, where the claim is that *at least one* of the disjuncts is true. Notice that this claim is still true when logic is both fun and hard. This is the type of disjunction represented by $\vee$.

So the above statement is symbolized as $L \vee H$.

Inclusive vs. Exclusive

However, 'or' (and 'unless') can be used in a logically different way:

> I will pass **or** fail logic.

This is ***exclusive*** **disjunction**, where the claim is that *exactly one* of the disjuncts is true. So this statement is more precisely stated as:

> I will pass **or** fail logic, **but not both**.

This is then symbolized differently:

$$(\mathrm{P} \underset{\text{or}}{\vee} \mathrm{F}) \underset{\text{but}}{\&} \underset{\text{not}}{\sim} (\underbrace{\mathrm{P} \mathbin{\&} \mathrm{F}}_{\text{both}}).$$

Implication

Recall that a **hypothetical statement** has the form of "if . . . then", asserting that whenever the "if" part is true, the "then" part must be true as well. It is symbolized using $\rightarrow$ (called "arrow").

So the hypothetical statement $p \rightarrow q$ asserts that if statement p is true, then statement q is true. In this example, p is the **antecedent** and q is the **consequent**.

Note: As you have probably figured out, the use of the lower-case, italic letters p and q means that *any* two generic statements can be connected together as antecedent and consequent within a hypothetical statement.

Implication: Basic Example

Consider the following hypothetical proposition:

> **If** I study hard **then** I pass the class.

Both antecendent and consequent are simple positive statements, which are symbolized:

> S: I **study** hard.
>
> P: I **pass** the class.

So the entire statement is symbolized as $\mathrm{S} \rightarrow \mathrm{P}$.

Note: Should I mention those upper-case, upright letters S and P?

Implication: Further Examples

Now there are a lot of *other* ways to express the *exact same logic* symbolized by $S \rightarrow P$:

If I study hard I pass the class.

My studying hard will **cause** me to pass the class.

I pass the class **if** I study hard.

Passing the class is a **necessary condition** for studying hard.

I study hard **only if** I pass the class.

Studying hard is a **sufficient condition** for passing the class.

These are the tricky ones to remember!

Necessary vs. Sufficient

Notice that "p **if** q" and "p is a ***necessary*** **condition** for q" is symbolized as $q \to p$. Necessary means that p is *required* (but may not be enough) to get q. The idea is that if you have q, then p was required to get you it. So if q then p, or $q \to p$.

However, "p **only if** q" and "p is a ***sufficient*** **condition** for q" is symbolized as $p \to q$. Sufficiency means that p is *enough* (but may not be required) to get q. The idea is that if you have enough p, then you have q. So if p then q, or $p \to q$.

Necessary vs. Sufficient

Consider the following hypothetical statement:

> Passing a **history** course is a *necessary* condition for earning a **degree** in journalism. ($D \rightarrow H$.)

The claim is that passing a history course is *required* to earn the degree, but passing it is *not enough*: you also have to pass a lot of other courses.

So the idea is that if you earn the degree (D), then you passed a history course (H), or $D \rightarrow H$.

Necessary vs. Sufficient

Consider the following hypothetical statement:

> Passing **logic** is a *sufficient* condition for fulfilling communication's **quantitative** course requirement. ($L \rightarrow Q$.)

The claim is that passing logic is *enough* to fulfill the requirement. But passing logic is *not required* to do so: passing statistics is an alternative for the requirement.

So the idea is if you pass logic (L), then you have fulfilled the quantitative course requirement (Q), or $L \rightarrow Q$.

If vs. Only If

The phrases "if" and "only if" work in similar ways.

"If" without an "only" indicates (as it does usually) that what follows is the *antecedent* of a hypothetical. In this way, it functions like the phrase "is a necessary condition for".

"Only if", however, indicates that what follows is the *consequent* of a hypothetical. As such, it is like the phrase "is a sufficient condition for".

Next Class...

We will learn how take apart statements and assess their truth value by using truth tables.

Also, please do not forget to turn in your response to the Lecture #10 Questionnaire on your way out.