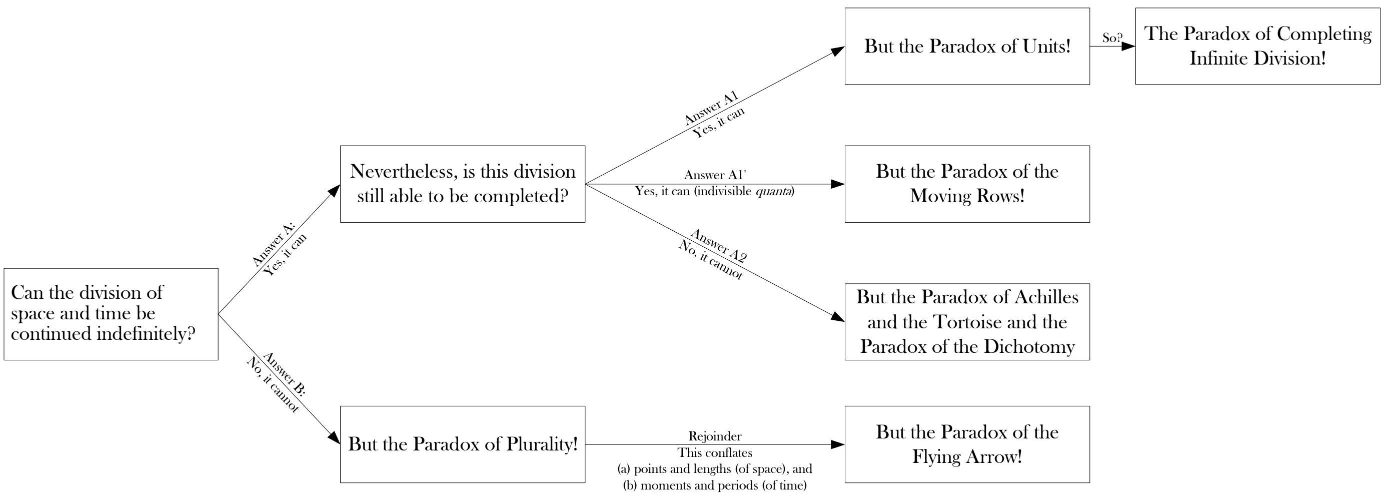
Zeno of Elea's Argument Against Plurality[1]

General Programme

There is no way to believe in plurality without leading to contradictions. Most people assume that individual things can be distinguished by breaking down the world (and any part of it) into its individual parts by spatial and temporal divisions. However, there is no way to divide reality into spatial and temporal parts without absurdity.

Schematic



The Question

Can the division of space and time be continued indefinitely? Either way, what do the resultant parts of such a division look like?

Answer A1: The division has an infinite number of steps, creating an infinite number of parts, but nevertheless, this division can still be completed

<u>Assumption of Divisibility</u>: If something has size, then it is divisible into smaller things. Conversely, if something is not divisible then it has no size.

<u>The Paradox of Units</u>: Units can have no size, yet they must have size

1. Units can have no size [From Simplicius *in Phys* 139.18-19]
    a. Assume they have size
    b. Then they are divisible into further parts
    c. This implies they are not units, but collection of units

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[1] This analysis of Zeno of Elea follows that of G.E.L. Owen's "Zeno and the Mathematicians", *Proceedings of the Aristotelian Society*, 58 (1957-58).

d. But they are units, so they must have no size

The upshot: When doing an exhaustive division of something, your end-products (even if infinite in number) cannot be further divided, but then they are indivisible and so have no size!

2. Units must have size [316a in KRS]
    a. Assume they have no size
    b. But if you add or subtract units to a thing, then you've added or subtracted nothing to the thing
    c. Therefore, units are nothing
    d. But the units are something, so they must have size

    The upshot: When doing an exhaustive division of something, you end-products should be able to be put together again to make things, but adding units without size together results in things without size!

The Paradox of Completing Infinite Division [316b in KRS]

1. An Example of Infinite Division
    a. Take something with some size
    b. Notice it has a front with some size
    c. And this front has a front with some size
    d. So there is an infinite division of parts here
    e. These are the "many things" here
2. The Paradox (Mirroring the Paradox of Units)
    a. The many things are so small without magnitudes
        i. The parts (units) have no size (if they had size, we have no completed the dividing process yet)
        ii. Then the parts are no-things, which is absurd
    b. The thing being divided is infinitely big
        i. The parts (units) have size (because units must have some size in order to be assembled into the whole thing)
        ii. There are infinitely many parts
        iii. Therefore the thing is infinitely big

Answer A2: The division has an infinite number of steps, creating an infinite number of parts, and this division can never be completed

The Paradox of Achilles and the Tortoise: "You cannot catch up"

1. The same general formula as the Paradox of Completing Infinite Division
2. Dividing up something into infinite parts, where there is always a "head" ready for dividing
3. But if the division can never be completed, how does Achilles ever catch up? (As experience shows us)

<u>The Paradox of the Stadium, the Race Course, or the Dichotomy</u>: “You can’t even get started”

1. The Achilles shows no last move, this shows no first move

Answer A1': This division can be completed because some things (“*quanta*”) that have size are indivisible (rejects the Assumption of Division)

<u>The Paradox of the Stadium or the Moving Rows</u>

1. Naïve Form
    a. In time *t*, the first *B* passes half the *A*s and all the *C*s
        i. When considering *A*, *B*’s speed is 2 units per *t*
        ii. When considering *C*, *B*’s speed is 4 units per *t*
        iii. Passing 2 *C*’s in the same time to pass 1 *A*
        iv. Therefore, the move which takes *t* also takes 2*t*
2. Sophisticated Form: Assume that each block is 1 *quanta* in size
    a. Then a thing can only pass 1 or 0 *A*/*B*/*C*, but not half of one
    b. But in passing 1 *C*, *B* passes ½ *A*
    c. Hence in passing 1 *C*, *B* actually passes 2 *C*s
        i. *B* can only pass all or nothing of an *A*
        ii. Since *B* is moving, it must have passed 1 *A*
        iii. And so *B* has also passed 2 *C*s
            1. Given that *B* is passing *C*s twice as fast as *A*s
    d. A contradiction!

Answer B: The division has a finite number of steps, with no further step logically possible (finite number of parts)

<u>The Paradox of Plurality</u> [315 in KRS]

1. If there are many existing things, these things are limited
    a. If there are many things, then there are units which can be used to count them
    b. Whatever the number of units of things that are, they must be the as many as they are
    c. So the number is a definite number (there is a limit, a *peiras*)
    d. This is the same thing as a finite number
    e. Therefore, the existing things are finite
2. If there are many things, these things are unlimited
    a. If there are (say) two distinct things A and B, they must be distinguishable by a third distinct thing C if A and B are to be two things and not one

b. But then A and C must be distinguishable by some thing D, A and D by E, and so forth for ever
c. Therefore, the existing things are infinite

<u>The Paradox of the Arrow</u>: “You cannot move at all”

1. Anything that occupies a space just its own size is stationary
2. In each moment of its flight, an arrow can only occupy such a space
3. Therefore, at each moment the arrow is not moving, but stationary
4. But what is true of the arrow at each moment of a period is true throughout the whole period
5. Hence during the whole flight, the arrow does not move, but is stationary