

Symplectic duality, nilpotent cones, and fixed points

Conical symplectic singularities provide a large class of Poisson varieties that give a geometric framework to study representation theory of many interesting non-commutative algebras. An important topic, extensively studied in the last 15 years, is the phenomenon of symplectic duality, establishing a connection between geometric properties of two different symplectic singularities. Lately, a systematic study of a more specific class of symplectic singularities was initiated. Namely, assume that a conical symplectic singularity X has a Poisson involution, or more generally, an action of a finite group of Poisson automorphisms. Then the fixed points with respect to this involution constitute a symplectic singularity, playing an important role in studying representation theory of algebras, such as twisted current Lie algebras, or i -quantum groups. The relation of taking fixed points to symplectic duality is, however, unclear. In my talk I will show how symplectic duality and the fixed-points construction naturally appear inside the nilpotent cones of semisimple complex groups, and what that suggests about the symplectic dual to the fixed-points construction.