

Hecke Traces, Braid Varieties, and Springer Actions

The HOMFLYPT polynomial is a link invariant that Jones–Ocneanu constructed using a family of traces on the Hecke algebras of the symmetric groups. It is categorified by triply-graded Khovanov–Rozansky (KhR) homology, a richer invariant that can be constructed by applying trace functors to complexes of Soergel bimodules, or equivalently, certain complexes of perverse sheaves on flag varieties. Seeking to make KhR more explicitly geometric, I introduced a generalized Steinberg variety for any positive braid, and showed that the KhR homology of its link closure is an isotypic summand of the cohomology of the variety under a Springer-type action. These Steinberg braid varieties are closely related to the twisted wild character varieties studied by Boalch–Yamakawa, and refine the Richardson braid varieties of recent interest in algebraic combinatorics. At least for periodic braids, they should be related via nonabelian Hodge theory to the homogeneous affine Springer fibers studied by Lusztig–Smelt, Sage, Sommers, and many others. I will explain where these varieties come from, and what predictions they offer for affine Springer theory.