Universal constructions for Poisson algebras. Applications.

We introduce the universal algebra of two Poisson algebras P and Q as a commutative algebra $A := \mathcal{P}(P, Q)$ satisfying a certain universal property. The universal algebra is shown to exist for any finite-dimensional Poisson algebra P and several of its applications are highlighted. For any Poisson P-module U, we construct a functor $U \otimes - : {}_A\mathcal{M} \to {}_Q\mathcal{PM}$ from the category of A-modules to the category of Poisson Q-modules which has a left adjoint whenever U is finite-dimensional. Similarly, if V is an A-module, then there exists another functor $- \otimes V : {}_P\mathcal{PM} \to {}_Q\mathcal{QM}$ connecting the categories of Poisson representations of P and Q and the latter functor also admits a left adjoint if V is finite-dimensional. If P is n-dimensional, then $\mathcal{P}(P) := \mathcal{P}(P, P)$ is the initial object in the category of all commutative bialgebras coacting on P. As an algebra, $\mathcal{P}(P)$ can be described as the quotient of the polynomial algebra $k[X_{ij} | i, j = 1, , n]$ through an ideal generated by $2n^3$ non-homogeneous polynomials of degree ≤ 2. Two applications are provided. The first one describes the automorphisms group $\mathrm{Aut}_{\mathrm{Poiss}}(P)$ as the group of all invertible group-like elements of the finite dual $\mathcal{P}(P)^{\circ}$. Secondly, we show that for an abelian group G, all G-gradings on P can be explicitly described and classified in terms of the universal coacting bialgebra $\mathcal{P}(P)$. Joint work with G. Militaru.