MTH 461/561 Due Tuesday, November 27
Fall 2018

Homework 9

We need the following definition.

Definition 1. Let G be a group acting on a set X by the map $\triangleright : G \times X \to X$. The *permutation representation* $\mathrm{Perm}_{\triangleright}$ attached to the action is the representation of G on the vector space with basis $\{e_x\}_{x \in X}$ defined by

$$ge_x = e_{g \triangleright x}, \ \forall g \in G, \ x \in X.$$

♦

Problem 1. *Let $H \leq G$ be an inclusion of finite groups. Show that the induced representation $\mathrm{Ind}_H^G \mathrm{triv}$ is isomorphic to the permutation representation obtained from the action of G on the set G/H of cosets.*

First, recall the following concept from class. In general, for a Young diagram λ, we denote by $\lambda_1 \geq \lambda_2 \geq \cdots$ the lengths of its rows.

Definition 2. Let λ and μ be two Young diagrams with the same number of boxes. A λ*-filling of* μ is a labeling of the boxes of μ with λ_1 copies of the symbol '1', λ_2 copies of the symbol '2', etc. so that the numbers increase weakly rightward along rows and strictly downward along columns. ♦

Problem 2. *Let λ and μ be two Young diagrams with the same number of boxes. Show that there is a λ-filling of μ if and only if the following conditions hold for all i simultaneously:*

$$\lambda_1 + \cdots + \lambda_i \leq \mu_1 + \cdots + \mu_i.$$

Problem 3. *Let λ be the partition $(n-2, 1, 1)$ of n and χ the character of the representation V_λ of S_n.*

If the cycle decomposition of $\sigma \in G$ has i_1 cycles of length one and i_2 cycles of length two, show that we have

$$\chi(\sigma) = \binom{i_1 - 1}{2} - i_2.$$