MTH 461/561 Due Thursday, November 01
Fall 2018

Homework 7

Problem 1. *Problem 4.12.1 from Etingof.*

Problem 2. *Exercise 3.10.1 from Etingof.*

For the next problem you'll need a definition.

Definition 1. Let A be a ring. An element $x \in A$ is *integral* if it satisfies some equation of the form

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

for some integers a_1 up to a_{n-1}. ♦

Remark 2. So in particular, the integral elements in $\mathbb{C}$ are exactly the algebraic integers we discussed in class. ♦

Problem 3. *Let A be a commutative ring and assume that as an abelian group it is isomorphic to $\mathbb{Z}^n$ for some n (i.e. it's a free abelian group of finite rank).*

Show that all elements of A are integral.

Recall that we used this in the course of the proof that for a G-irrep V we have

$$\dim V \mid |G|.$$

We applied this principle to the ring $A \subset \mathbb{Z}G$ generated by the elements

$$\sum_{g \in C} g$$

where $C \subset G$ were conjugacy classes.

You can use whatever you want about the structure of finitely generated abelian groups. For instance, you can use the fact that every subgroup of $\mathbb{Z}^n$ is isomorphic to $\mathbb{Z}^m$ for some $m \leq n$.