MTH 461/561 Due Thursday, September 20
Fall 2018

Homework 2

Problem 1. *Let A be an algebra over the complex numbers and $T : V \to W$ a module morphism between A-modules.*

(a) *Show that*
$$\ker T := \{v \in V \mid Tv = 0\}$$
is a submodule of V.

(b) *Show that*
$$\operatorname{Im} T := \{w \in W \mid w = Tv \text{ for some } v \in V\}$$
is a submodule of W.

Problem 2. *2.3.15 in Etingof's book:*

- *just the first part (i.e. disregard the last sentence) if you're in 461;*
- *all of it if you are in 561.*

Problem 3. *Problem 2.3.16 (a) from Etingof's book.*

Remember that for us the fields k Etingof refers to are always just $\mathbb{C}$, the field of complex numbers.

Problem 4. *Let n be a positive integer. This problem is supposed to guide you through the proof of the fact that the algebra $M_n(\mathbb{C})$ is simple. Throughout, let I be a non-zero ideal of $M_n(\mathbb{C})$.*

(a) *Consider the matrix E_{ij} having 1 as its (i, j) entry and all other entries zero. Show that I contains E_{ij} for some choice of (i, j).*

(b) *Show that you then have $E_{ij} \in I$ for all choices of (i, j) (use the equations $E_{ij}E_{jk} = E_{ik}$, etc.).*

(c) *Conclude that $I = M_n(\mathbb{C})$.*