MTH 461/561 Due Thursday, October 11
Fall 2018

Exam 1

Problem 1. *Let V and W be two finite-dimensional representations of a finite group. Show that $W \otimes V^*$ contains a copy of the trivial representation if and only if there is an irrep S of G that appears as a summand in both V and W.*

Problem 2. *Problem 4.5.2 in Etingof's book.*

Problem 3. *Let G be a finite group and V a finite-dimensional representation. The complex conjugate representation $\overline{V}$ is defined as follows:*

- *As a vector space, $\overline{V}$ has the same underlying set and addition operation as V. We denote the element of $\overline{V}$ that is identical to $v \in V$ by $\bar{v}$, to clarify which ambient vector space we're in.*
- *The complex scalar action on $\overline{V}$ is the old one twisted by complex conjugation: if $c \in \mathbb{C}$ and $v \in V$, then*
$$c\bar{v} = \overline{\bar{c}v} \in \overline{V}$$
where the bar on top of c indicates complex conjugation.
- *The G-action is as before: for $\bar{v} \in \overline{V}$ we have*
$$g\bar{v} = \overline{gv}.$$

Prove that $\overline{V}$ is isomorphic to the dual representation V^*.

Problem 4. *Find the character table of the group G with generators σ and g subject to relations*
$$\sigma^2 = 1,\ g^5 = 1,\ \sigma g \sigma^{-1} = g^{-1}.$$
You can take it for granted that
$$G = \{g^i \sigma^j \mid 0 \le i \le 4,\ 0 \le j \le 1\}.$$