Application 4.1 Approximating Piecewise Linear Functions

Background

• Numerous Applications with different scientific fields use piecewise linear functions.
• These functions contain a large number of breakpoints.
• They are hence very expensive to store, manipulate and even to evaluate.
• Approximating Piecewise Linear Function Saves on
• Storage Space.
• Cost of using the function
Application 4.1 Approximating Piecewise Linear Functions

We define-
• $F_1(x)$- piecewise linear function of scalar ‘x’
• It passes through ‘n’ points where $a_1 = (x_1, y_1), \ldots, a_n$
• The function varies linearly between every two consecutive points $x_i$ and $x_{i+1}$
• We consider situations in which $n$ is very large and for practical reasons

We wish to approximate the function $F_1(x)$ by another function $F_2(x)$ that passes through only a subset of the points $a_1, \ldots, a_n$
Application 4.1  Approximating Piecewise Linear Functions

We will formulate the problem –

• As the shortest path problem on a network G with n nodes.

• The network contains an arc(i,j) for each pair of nodes i and j such that i<j

We define Total Cost

\[ c_{ij} = \alpha + \beta \left[ \sum (f_1(x_k) - f_2(x_k))^2 \right] \]