Problem 1.
We have
\[ P(B) = 1 - P(B^c) = 1 - 0.35 = 0.65. \]
Also, by rearranging the formula
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B), \]
we obtain
\[ P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.55 + 0.65 - 0.75 = 0.45. \]

Problem 5.
We have
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1 - P(A^c) + P(B) - P(A \cap B) = 1 - 0.6 + 0.3 - 0.2 = 0.5. \]
Problem 8.

We claim that the optimal order is to play the weakest player second (the order in which the other two opponents are played makes no difference). To see this, let $p_i$ be the probability of winning against the opponent played in the $i$th turn. Then you will win the tournament if you win against the 2nd player (prob. $p_2$) and also you win against at least one of the two other players [prob. $p_1 + (1 - p_1)p_3 = p_1 + p_3 - p_1p_3$]. Thus the probability of winning the tournament is

$$p_2(p_1 + p_3 - p_1p_3).$$

The order $(1, 2, 3)$ is optimal if and only if the above probability is no less than the probabilities corresponding to the two alternative orders:

$$p_2(p_1 + p_3 - p_1p_3) \geq p_1(p_2 + p_3 - p_2p_3),$$

$$p_2(p_1 + p_3 - p_1p_3) \geq p_3(p_2 + p_1 - p_2p_1).$$

It can be seen that the first inequality above is equivalent to $p_2 \geq p_1$, while the second inequality above is equivalent to $p_2 \geq p_3$.

Problem 10.

Since the events $A \cap B^c$ and $A^c \cap B$ are disjoint, we have using the additivity axiom repeatedly,

$$P((A \cap B^c) \cup (A^c \cap B)) = P(A \cap B^c) + P(A^c \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B).$$
Let $B$ be the event that Bob tossed more heads. Let $X$ be the event that after each has tossed $n$ of their coins, Bob has more heads than Alice, let $Y$ be the event that under the same conditions, Alice has more heads than Bob, and let $Z$ be the event that they have the same number of heads. Since the coins are fair, we have $P(X) = P(Y)$, and also $P(Z) = 1 - P(X) - P(Y)$. Furthermore, we see that

$$P(B | X) = 1, \quad P(B | Y) = 0, \quad P(B | Z) = \frac{1}{2}.$$ 

Now we have, using the theorem of total probability,

$$P(B) = P(X) \cdot P(B | X) + P(Y) \cdot P(B | Y) + P(Z) \cdot P(B | Z)$$

$$= P(X) + \frac{1}{2} \cdot P(Z)$$

$$= \frac{1}{2} \cdot (P(X) + P(Y) + P(Z))$$

$$= \frac{1}{2} \cdot (1)$$

as required. What is happening here is that Alice’s probability of more heads than Bob is less than $1/2$, so Bob has an advantage. However, the probability of equal number of heads is positive, and when added to Alice’s probability of more heads, it gives $1/2$. 
Problem 21

For convenience, we will number each of the parking spaces. We will draw a sequential probability tree to illustrate the sample space:

Mary can choose any of the $n$ parking spaces. She has a probability of $1/n$ of selecting any particular space. Tom can choose any of the remaining $n - 1$ spaces and has a probability of $1/(n - 1)$ of choosing any particular space (other than the one Mary chose).

There are $n(n - 1)$ leaves on the tree, and each leaf is equally likely to occur. When we look at the leaves on the branches where Mary does not choose spaces $1, 2, n - 1$, or $n$, we see that 4 leaves on each of these branches is in our event (two spaces on each side of Mary's car). When Mary chooses spaces $2$ or $n - 1$, there are three such leaves (one space on one side and two on the other side). When Mary chooses spaces $1$ or $n$, there are only two such leaves (she is at one end or the other of the parking lot).

Therefore, the probability that they are parked within two spaces of each other is:

$$P(A) = \frac{(4)(n - 4) + (3)(2) + (2)(2)}{n(n - 1)} = \frac{4n - 6}{n(n - 1)}$$

Note that for $n = 2$ and $n = 3$, $P(A) = 1$, as expected.
Problem 35

(a) $2^{25}$.

(b) First note that under this rule, each match will be stopped after a number of games ranging from 13 to 25. If a match will be stopped at the $k$'th game with player 1 having 13 points, then the last game was a win and $k - 13$ of the previous games was a loss. So, there are $\binom{k - 1}{13}$ matches that ends at the $k$'th game with player 1 having a score of 13. Taking player 2 into consideration and summing over $k$, we obtain

$$2 \sum_{k=13}^{25} \binom{k - 1}{k - 13}$$

possible distinct score sequences.