HW # 1: MTH 418/518 Partial Differential Equations

Do exercises in *Partial Differential Equations: An Introduction by Walter Strauss*:

1. Section 1.1
   # 3, 4, 11

2. Section 1.2
   # 1, 3, 8, 11

3. Section 1.4
   # 3

4. Consider the following Burgers’ equation for $u : [-1, 1] \times (0, T] \rightarrow \mathbb{R}$, $T > 0$,

   \[ u_t + \left( \frac{1}{2} u^2 \right)_x = 0. \]

   a) Suppose that the initial condition is given by
   \[ u(x, 0) = -\sin(\pi x). \]
   Sketch the characteristic curves in $(x, t)$ plane.
   b) Find the exact solution (in implicit form).
   c) Sketch the series of solutions with different time $t$.
   d) At what time $t_s$, two characteristic lines first meet together?
   e) (optional) Using b), can you plot the exact solution $u(x, t)$ for $t < t_s$?