**Steps** to use the chain rule:

1. Decompose the given function $y$ into a composition of two functions:
   $$x \rightarrow u \rightarrow y$$
   where $u$ is a function of $x$ (called the inside function) and $y$ is a function of $u$ (called the outside function).
2. Calculate $\frac{dy}{du}$, the derivative of the outside function $y(u)$, with the inside function $u(x)$ plugged in for $u$.
3. Calculate $\frac{du}{dx}$, the derivative of the inside function $u(x)$.
4. The chain rule: the derivative $\frac{dy}{dx}$ is $\frac{dy}{du} \cdot \frac{du}{dx}$, the product of the two step-derivatives you found above.

**Example** Differentiate $y = (x^4 + 3x^2)^3$

**Solution**

1. Find the composition structure. Here $y$ is the cubic power of $x^4 + 3x^2$. So the term $x^4 + 3x^2$ sits inside the cubic power function. So naturally we can identify:
   - Inside function $u = x^4 + 3x^2$, Outside function $y = u^3$.
2. Calculate $\frac{dy}{du}$: Focus on the outside function $y = u^3$, and treat $y$ as a function of the independent variable $u$. Then apply the power rule to this function, and then plug in the inside function for $u$:
   $$\frac{dy}{du} = 3u^2$$
   $$\big|_{u = x^4 + 3x^2} = 3(x^4 + 3x^2)^2$$
3. Calculate $\frac{du}{dx}$: Focus on the inside function $u = x^4 + 3x^2$. Its derivative is
   $$\frac{du}{dx} = 4x^3 + 6x$$
4. Then by the chain rule, the derivative of $y$ with respect to $x$ is
   $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3(x^4 + 3x^2)^2 \cdot (4x^3 + 6x)$$

**Example** Differentiate $y = \frac{1}{x^2 + 3x + 1}$

**Solution**

1. Find the composition structure. Notice that the function is the reciprocal of $x^2 + 3x + 1$. So we can take the part of denominator $x^2 + 3x + 1$ to be the inside, and the reciprocal part to be the outside.
   - Inside function $u = x^2 + 3x + 1$, Outside function $y = \frac{1}{u} = u^{-1}$.
2. Calculate $\frac{dy}{du}$: Focus on the outside function $y = u^{-1}$, and treat $y$ as a function of the independent variable $u$. Then apply the power rule to this function, and then plug in the inside function for $u$:
   $$\frac{dy}{du} = (-1)u^{-2}$$
   $$\big|_{u = x^2 + 3x + 1} = (-1) \cdot (x^2 + 3x + 1)^{-2}$$
3. Calculate $\frac{du}{dx}$: Focus on the inside function $u = x^2 + 3x + 1$. Its derivative is

$$\frac{du}{dx} = 2x + 3$$

4. Then by the chain rule, the derivative of $y$ with respect to $x$ is

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (-1) \cdot (x^2 + 3x + 1)^{-2} \cdot (2x + 3)$$

**Example** Differentiate $y = \frac{1}{\sqrt{5x-x^2}}$

**Solution**

1. Inside function $u = 5x - x^2$, and the outside function $y = \frac{1}{\sqrt{u}} = u^{-1/2}$.

2. Calculate $\frac{dy}{du}$ by power rule: $\frac{dy}{du} = \frac{-1}{2}u^{-3/2}$ plug in $u=5x-x^2$, $\frac{1}{2} \cdot (5x-x^2)^{-3/2}$

3. Calculate $\frac{du}{dx} \cdot \frac{du}{dx} = 5 - 2x$

4. By the chain rule, the derivative of $y$ with respect to $x$ is

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{-1}{2} \cdot (5x-x^2)^{-3/2} \cdot (5-2x)$$