Information

Due: 09/23 in Lecture
Please staple your homework

Problems

1 Limit definition of derivative. Let $f(x) = x^2 - 2x + 3$.
   (1) Evaluate $f(x+h)$, as an expression of $x$ and $h$;
   (2) Simplify the differential quotient $\frac{f(x+h) - f(x)}{h}$, until the $h$ in the denominator is canceled;
   (3) Take the limit of the reduced quotient in part (2) as $h \to 0$, to obtain the derivative function $f'(x)$;
   (4) Use the derivative function $f'(x)$ to find the equation of the tangent line of $f(x)$ at $x = 3$.

2 Limit definition of derivative. (1) Use the limit definition to compute the derivative function of $f(x) = 3 - 4x - 2x^2$;
   (2) Find the equation of the tangent line of $f(x)$ at $x = 1$;
   (3) Is $f(x)$ increasing, decreasing or a critical point at $x = 1$? why?
   (4) Find the equation of the tangent line of $f(x)$ at $x = -1$;
   (5) Is $f(x)$ increasing, decreasing or a critical point at $x = -1$? why?

3 Limit definition of derivative. (1) Use the limit definition to compute the derivative function of $f(x) = x^3 - 2$. (Hint: you may use the cube identity: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.)
   (2) Find the equation of the tangent line of $f(x)$ at $x = 1$;
   (3) Is $f(x)$ increasing, decreasing or a critical point at $x = 1$? why?

4 Limit definition of derivative. (1) Use the limit definition to compute the derivative function of $f(x) = 12x - x^3$.
   (Hint: you may use the cube identity: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.)
   (2) Find the equation of the tangent line of $f(x)$ at $x = 1$;
   (3) Is $f(x)$ increasing, decreasing or a critical point at $x = 1$? why?
   (4) Which of the following $x$-values are critical points of $f(x)$? why?
      (A) $x = -2$; (B) $x = -1$; (C) $x = 0$; (D) $x = 1$; (E) $x = 2$.

5 Derivative rules. Use the derivative rules to compute the derivative of the following functions, and then find the equation of the tangent line at $x = 2$.
   (1) $y = x^2 - 6x + 2$; (2) $f(x) = \frac{x^2}{3} + \frac{1}{4}$; (3) $f(x) = \frac{x^4}{2} - 3x + \frac{1}{3}$

6 Derivative rules and first derivative test. (1) Use the derivative rules to compute the derivative of $f(x) = x^2 - 4x + 3$.
   (2) Fill out the following table to discuss the property of $f(x)$ at the given values:

<table>
<thead>
<tr>
<th>$x$-value</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope of tangent line at this $x$</td>
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<td></td>
<td></td>
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<tr>
<td>Direction of tangent line (inc/dec/horizontal) at $x$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$f(x)$ is an inc/dec/critical point at this $x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(3) Plot the sample points labeled in part (2). Then sketch a rough graph of \( f(x) \), showing the local properties you had from part (2).

7 **Derivative rules and first derivative test.** (1) Use the derivative rules to compute the derivative of \( f(x) = x^3 - 3x \).

(2) Fill out the following table to discuss the property of \( f(x) \) at the given values:

<table>
<thead>
<tr>
<th>x-value</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope of tangent line at this ( x )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Direction of tangent line (inc/dec/horizontal) at ( x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) ) is an inc/dec/critical point at this ( x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(3) Plot the sample points labeled in part (2). Then sketch a rough graph of \( f(x) \) over the domain \([-2, 2]\), showing the local properties you had from part (2).

8 **Marginal analysis: Cost function.** Suppose that the cost (in dollars) for a company to produce \( x \) pairs of a new line of jeans is

\[
C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3
\]

(1) Compute the marginal cost function \( C'(x) \);

(2) Compute value \( C'(100) \). Then interpret the meaning of \( C'(100) \) as the approximate change of revenue if the price increases or decreases by \$ 1.

(3) Compute the total cost of producing 100 pairs and 101 pairs, and then find the actually additional cost of producing the 101st pair. (Then compare the answer with \( C'(100) \) in part (2). Are they close?)

(4) Compute the total cost of producing 100 pairs and 99 pairs, and then find the change of cost if the production reduces from 100 pairs to 99 pairs.

(5) Compute the value \( R'(150) \) and interpret the meaning.

(6) Compute the actual additional cost of producing the 151st pair and compare it with \( R'(150) \) you had in part (5).

9 **Marginal analysis: Revenue.** A store has selling 400 DVD players a week at \$350 each. A market surveys shows that for each \$ 10 discount offered to buyers, the number of players sold will increase by 20 a week.

(1) Assume the demand function is linear. Let \( x \) be the price and \( Q \) be the quantity of player sold. Establish the demand function \( Q(x) \).

(2) Let \( R \) (in dollars) be the revenue of the store from selling the players at price \( x \). Express \( R \) as a function of \( x \).

(3) Find the marginal revenue \( R'(x) \).

(4) Compute the value \( R'(350) \). Interpret this value as the approximate change of revenue if the price increases or decreases by \$ 1.

(5) From the current price \$350, to increase the revenue, should the store raise or drop the unit price of the player? Use the marginal revenue to answer this question.

(6) Compute the actual change of revenue for the option of raising price from 350 to 351 by \$ 1, and for the option of dropping price from 350 to 349 by \$ 1. Is the choice you made in part (5) correct?

(7) Suppose now the unit price is \$ 250. Compute the value \( R'(250) \) and interpret this value.

(8) From the current price \$250, to increase the revenue, should the store raise or drop the unit price of the player? Use the marginal revenue to answer this question.

(9) Compute the actual change of revenue for the option of raising price from 250 to 251 by \$ 1, and for the option of dropping price from 250 to 249 by \$ 1. Is the choice you made in part (8) correct?
Answer to Homework 3

Error If you find any error, please tell me: yinsu@buffalo.edu. Bonus points will be rewarded.

1. (1) $f(x + h) = (x + h)^2 - 2(x + h) + 3 = x^2 + 2xh + h^2 - 2x - 2h + 3$; (2) $2x + h - 2$;
   (3) $f'(x) = 2x - 2$; (4) $y - 6 = 4(x - 3)$ or $y = 4x - 6$.

2. (1) $f'(x) = -4 - 4x$; (2) $y + 3 = -8(x - 1)$ or $y = -8x + 5$;
   (3) Decreasing because $f'(1) = -8$ is negative; (4) $y - 5 = 0(x + 1)$ or $y = 5$; (5) Critical point, because $f'(-1) = 0$.

3. (1) $f'(x) = 3x^2$; (2) $y + 1 = 3(x - 1)$ or $y = 3x - 4$; (3) Increasing.

4. (1) $f'(x) = 12 - 3x^2$; (2) $y - 11 = 9(x - 1)$ or $y = 9x + 2$; (3) Increasing; (4) (A) and (E).

5. (1) $y' = 2x - 6$ and $y'(2) = -2$; Tangent line: $y + 6 = -2(x - 2)$; (2) $f'(x) = \frac{2}{3}x$ and $f''(2) = \frac{4}{3}$; Tangent line
   $y - \frac{19}{12} = \frac{4}{3}(x - 2)$; (3) $f'(x) = 2x^3 - 3$ and $f''(2) = 13$; Tangent line $y - \frac{7}{3} = 13(x - 2)$

6. (1) $f'(x) = 2x - 4$;

<table>
<thead>
<tr>
<th>$x$-value</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-2</td>
<td>0</td>
<td>3</td>
</tr>
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<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
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<td>Dec</td>
<td>Horizontal</td>
<td>Inc</td>
<td>Inc</td>
</tr>
<tr>
<td>$f(x)$ is an inc/dec/CP at this $x$</td>
<td>Dec</td>
<td>Dec</td>
<td>CP</td>
<td>Inc</td>
<td>Inc</td>
</tr>
</tbody>
</table>

(3) See the graph

7. (1) $f'(x) = 3x^2 - 3$;

<table>
<thead>
<tr>
<th>$x$-value</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$-value</td>
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<td>2</td>
<td>0</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>slope of tangent line at this $x$</td>
<td>Inc</td>
<td>Horizontal</td>
<td>0</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>Direction of tangent line at $x$</td>
<td>Inc</td>
<td>Horizontal</td>
<td>Dec</td>
<td>Horizontal</td>
<td>Inc</td>
</tr>
<tr>
<td>$f(x)$ is an inc/dec/CP at this $x$</td>
<td>Inc</td>
<td>CP</td>
<td>Dec</td>
<td>CP</td>
<td>Inc</td>
</tr>
</tbody>
</table>

(3) See the graph

8. (1) $C'(x) = 3 + 0.02x + 0.0006x^2$;

(2) $C'(100) = 11$; The additional cost to produce the 101st pair is approximately 11 dollars;

(3) $C(100) = 2600$ and $C(101) = 2611.07$. The actual additional cost is $C(101) - C(100) = 11.07$ dollars, which is close to the marginal cost $C'(100)$ obtained in part (2).

(4) $C(100) = 2600$ and $C(99) = 2589.07$. The change of cost of reducing the production by 1 is $C(99) - C(100) = 10.93$.

(5) $C'(150) = 19.5$; The additional cost to produce the 151st pair is approximately 141 dollars;

(6) $C(150) = 3350$ and $C(151) = 3369.6$. The actual additional cost is $C(151) - C(150) = 19.6$, which is close to the marginal cost $C'(150)$ obtained in part (4).

9. (1) $Q(x) = -2x + 1100$; (2) $R(x) = -2x^2 + 1100x$; (3) $R'(x) = -4x + 1100$;

(4) $R'(350) = -300$. If the price goes up by $1 from 350, approximately the revenue will drop by 300 dollars. If the price goes down by $1 from 350, the revenue will increase approximately 300 dollars.

(5) The store should drop the price from $350.

(6) The actual revenue change for raising the price by $1 is $R(351) - R(350) = -302$ dollars; The actual revenue change for dropping the price by $1 is $R(349) - R(350) = 298$ dollars. Thus dropping price is the right choice.

(7) $R'(250) = 100$. If the price goes up by $1 from 250, approximately the revenue will increase by 100 dollars, or equivalently, if the price goes down by $1 from 250, approximately the revenue will decrease by 100 dollars.

(8) At price $250, the store should raise the price.

(9) Raise price by 1: revenue change is $R(251) - R(250) = 98$; Drop price by 1: revenue change is $R(249) - R(250) = -102$. Thus raising the price is the right choice.