Learning objects:

1. How to evaluate a function at a number or an expression.
2. The slope-intercept and the slope-point formula of a straight line. Know how to interpret the sign and value of the slope.
3. Graphical and limit definition of the derivative. Use the limit definition to calculate the derivative function.
4. Use derivative rules to differentiate polynomials: constant rule, power rule, sum/difference rule and constant-multiple rule.
5. Know how to find the equation of tangent line using the derivative rules.
6. Know the local classification of points in a function, and how the first derivative test connects the sign of the derivative and the shape of the \( f(x) \)-curve.
7. Know how to find critical points and use the first derivative test to determine the increasing/decreasing intervals and maximal/minimal points. Then graph the function.
8. Set up simple cost, demand and revenue function in the linear model.
9. Interpret the derivative as rate of change. Marginal cost and marginal revenue.
10. Use critical points and first derivative test to find the maximal revenue.

Points: Regular problems: 60 points. Bonus problem: 10 points.

What to bring and to get:

1. Needed: Pen/Pencil, Photo ID.
3. Not allowed: Textbook, notes, other references.
4. You will get in exam: a formula sheet. (See next page. You don’t need to print it out.)

Practice:

1. A practice exam is attached. Answer key is provided.
2. Homework coverage: Homework 1 to 4.

Remarks:

1. If you need any special accommodation, please tell me ASAP.
2. You need an official excuse if you miss the exam and want to make it up. The makeup exam will be harder than the usual one.
• **Limit definition of the derivative:** \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \).

• **Basic differentiation rules**
  
  - Constant rule: \((\text{constant})' = 0\)
  
  - Power rule: \((x^n)' = nx^{n-1}\)
  
  - Sum and difference rule: \((f(x) + g(x))' = f'(x) + g'(x)\)
  
  - Constant-multiple rule: \((k \cdot f(x))' = k \cdot f'(x)\)

• **Slope-point formula:** The equation of the line through a given point \((x_1, y_1)\) with slope \(m\) is \(y - y_1 = m(x - x_1)\).

• **Quadratic formula:** The roots of the quadratic equation \(ax^2 + bx + c = 0\) are \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\).
1. (15 points) (1) Compute the derivative of the following functions:
   
   (a) \( f(x) = 6x^4 - 3x^2 + 2 \);  
   (b) \( f(x) = \frac{3x^3}{4} + \frac{x}{2} - \frac{1}{3} \).

2. (15 points) Let \( f(x) \) be the function \( f(x) = \frac{3x^4}{4} - 4x^3 + 6x^2 - 2 \).
   
   (1) Find all critical points of \( f(x) \).
   (2) Use the derivative test to find all intervals where \( f(x) \) is increasing/decreasing.
   (3) Classify all critical points.
   (4) Sketch a rough graph of \( f(x) \) in the \( xy \)-plane to show all critical points and the properties you obtained from the first derivative test.

3. (15 points) A travel agency offers a cruise of Caribbean islands. At the cost \$1200 per person, 400 people would take the trip. Each \$20 reduction of the price would bring in 40 more passengers. Let \( x \) be the cruise price, and \( Q \) be the number of passengers in the cruise, and \( R \) be the revenue for the travel agency.
   
   (1) Assume the demand is linear. Express \( Q \) as a function of \( x \).
   (2) Express the revenue \( R \) as a function of the price \( x \).
   (3) Compute \( R'(1200) \) and interpret the derivative value in terms of the change of revenue. At the price level \$1200, to increase the revenue, should the agency increase or decrease the cruise price.
   (4) Use the derivative test to show the optimal price where the revenue achieves the maximum. Also compute the maximal revenue.

4. (15 points) (1) The following figures show graph of functions. Answer the questions

   ![Graphs](image)

   (I) (II) (III) (IV)

   (a) Which function(s) have a positive \( f'(x) \) for all \( x \)?
   (b) Which function(s) have critical points?
   (c) Which function(s) have a negative derivative value at the \( y \)-intercept?
(2) Assume we know that the derivative of \( f(x) \) is \( g(x) \), and the derivative of \( g(x) \) is \( h(x) \), and the following figure shows their graphs. Match the functions \( f(x), g(x), h(x) \) with the labeled curves (I), (II), (III).

(3) Does there exist a polynomial whose smooth curve has two local maximal points but no local minimal point. If yes, sketch such a curve, if no, explain why.

(4) Determine if each of the following statements is true or false:

(a) If \( f'(x) \) is increasing, then \( f(x) \) is positive.
(b) All maximal and minimal points are critical points.
(c) The points (1, 2), (−1, 0) and (4, 5) lie in a common straight line.

Bonus (Bonus Problem. 10 points) Let \( f(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)(x-8)(x-9)(x-10) \). Compute the value of \( f'(0) \), the derivative of \( f(x) \) at \( x = 0 \).

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**Answer**

If you find any error, please tell me: yinsu@buffalo.edu. Bonus points will be rewarded.

1. (a) \( f'(x) = 30x^4 - 6x \); (b) \( f'(x) = \frac{9x^2}{4} + \frac{1}{2} \); (2) \( y = 8x - 11 \).
2. (1) \( f'(x) = 3x^3 - 12x^2 + 12x \). Critical points: \( x = 0 \) and \( x = 2 \).
   (2) Interval of increase: \( (0, +\infty) \); Interval of decrease: \( (-\infty, 0) \).
   (3) Min: \( (0, -2) \); Increasing critical point: \( (2, 2) \).
   (4) See the graph.
3. (1) \( Q = -2x + 2800 \); (2) \( R = -2x^2 + 2800x \); (3) \( R'(1200) = -2000 \); If the price increases from \$ 1200 to \$1201, the revenue will drop by approximately \$ 2000. The agency should drop the price. (4) Optimal price: \$ 700; Maximal revenue: \$ 980000.
4. (1) (II) and (IV); (b) (I) and (III); (c) (I).
   (2) \( f(x) \) is curve (III); \( g(x) \) is curve (II) and \( h(x) \) is curve (I).
   (3) No. The sign of the derivative around a maximal point changes from + to 0 to −. Thus between two maximal points, the sign of \( f'(x) \) has to continuously change from − to +. In this process \( f'(x) \) has to pass the zero value, which gives a minimal point.
   (4) (a) False; (b) True; (c) True.

**Bonus**

\[ f'(0) = (-1)(-2)(-3) \cdots (-10) = 3628800. \] (Hint: take \( x = 0 \) in the limit definition of the derivative.)