Problems:

1. Use product rule to compute the derivative of the following functions:
   (1) $f(x) = (x^2 + 1)(x^2 - 1)$; (2) $f(x) = (2x - \sqrt{x})(x^2 - \frac{3}{x})$.

2. Use quotient rule to compute the derivative of the following functions:
   (1) $f(x) = \frac{x + 1}{x - 1}$; (2) $f(x) = \frac{3x^2 - 2}{\sqrt{x} + 3x}$.

3. Given $f(x) = \frac{x^3 - 2x - 4}{x^2}$, we will compute the derivative of $f(x)$ in different ways:
   (1) Use quotient rule;
   (2) Rewrite $f(x) = (x^3 - 2x - 4) \cdot \frac{1}{x^2} = (x^3 - 2x - 4) \cdot x^{-2}$ and use product rule;
   (3) Distribute the numerator over the denominator: $f(x) = \frac{x^3}{x^2} - \frac{2x}{x^2} - \frac{4}{x}$ and differentiate without using product rule or quotient rule.
   (4) Simplify the answers you get above and show that all three methods give the same result.

4. Differentiate the following functions using appropriate derivative rules
   (1) $f(x) = 5x - x^2(x^4 - 3x) + \frac{2}{x}$; (2) $f(x) = \frac{5}{\sqrt{x^2 + 1}} + (x - 1)(x + 1)$;
   (3) $f(x) = \frac{x}{x + \frac{1}{x}}$; (4) (optional) $f(x) = \frac{1}{x - 1}$.

5. Compute the first, second, third derivative of $f(x) = \sqrt{x}$, and then evaluate $f(4)$, $f'(4)$, $f''(4)$ and $f'''(4)$.

6. (Optional) Notice that if we differentiate $f(x) = x^3$, we will see that the higher derivatives are
   
   $f'(x) = 3x^2$ \hspace{5mm} $f''(x) = 6x$ \hspace{5mm} $f'''(x) = 6$ \hspace{5mm} $f^{(4)}(x) = 0$ \hspace{5mm} $f^{(5)}(x) = 0$, \hspace{5mm} $f^{(6)}(x) = 0 \cdots$

   Since the derivative of 0 is 0, we see that after we differentiate $f(x)$ for 4 times, the higher derivatives will all become zero.
   (1) Try to keep differentiating $f(x) = 2x^4$ until the higher derivatives become 0.
   (2) Try to keep differentiating $f(x) = \frac{1}{x}$. Will the higher derivatives stabilize at 0 after some steps?

7. A particle is moving in a straight line in such a way such that its position at time $t$ (in seconds) is $s(t) = t^2 + 3t + 2$ feet to the origin.
   (1) What is the initial position when $t = 0$?
   (2) What is the initial velocity of this particle at $t = 0$?
   (3) What is the velocity of the object when the time is 5 seconds.
   (4) When will the object be 6 feet from the origin?
   (5) At what velocity will the object be when it is 6 feet from the origin?
8. A toy rocket fired straight up into the air has height function \( f(t) = 160t - 16t^2 \) feet after \( t \) seconds.

1. What is the velocity at \( t = 0 \)?
2. What is the velocity after 4 seconds and after 7 seconds?
3. Is the rocket moving upward or downward after 4 seconds and after 7 seconds?
4. When will the rocket arrive the peak before it drops down?
5. When will the rocket hits the ground?
6. When the rocket hits the ground, what is its velocity?

9. Given \( f(x) = \sqrt{x} \).

1. Compute \( f'(x) \) and then evaluate \( f'(4) \).
2. Note that \( f(4) = 2 \). Use (1) to approximate \( \sqrt{4.01} \).

10. Given \( f(x) = \frac{1}{x} \). Use the fact that \( f(1) = 1 \) to estimate \( \frac{1}{1.02} \).

### Homework 4 Answer

1. (1) \( f'(x) = 2x(x^2 - 1) + 2x(x^2 + 1) \); (2) \( f'(x) = (x - \frac{1}{2}x^{-1/2})(x^2 - \frac{3}{2}) + (2x - \sqrt{x})(2x + 3x^{-2}) \).
2. (1) \( f'(x) = -\frac{2}{(x-1)^2} \); (2) \( f'(x) = \frac{6x(\sqrt{x} + 3x) - (3x^2 - 2x)(1/2x^{-1/2}) + 3}{(\sqrt{x} + 3x)^2} \).
3. Omitted
4. (1) \( f'(x) = 5 - 2x(x^4 - 3x) - x^2(4x^3 - 3) - 3x^{-2} \); (2) \( f'(x) = \frac{-5/2x^{-1/2}}{(\sqrt{x} + 2)^2} \); (3) \( f'(x) = \frac{x + x^{-1} - x(1 - x^{-2})}{(x + x^{-1})^2} \);
5. \( f'(x) = \frac{1}{2}x^{-1/2} \) \( f''(x) = -\frac{1}{4}x^{-3/2} \) \( f'''(x) = \frac{3}{8}x^{-5/2} \).
6. (1) The higher derivative becomes zero since \( f^{(5)}(x) \), the fifth derivative.
7. (2) The higher derivatives will never stabilize at 0.
8. (1) 2; (2) 3; (3) 13; (4) after 1 second; (5) 5.
9. (1) \( f'(x) = \frac{1}{2}x^{-1/2} \) and \( f'(4) = \frac{1}{4} \); (2) \( \sqrt{4.01} = f(4.01) \approx f(4) + f'(4) \cdot 0.01 = 2.0025 \).
10. \( \frac{1}{1.02} \approx 0.98 \).