Problems:

1. Given a piecewise function defined as

\[ f(x) = \begin{cases} 
  x^2 + 2x - 3, & \text{if } x > 1 \\
  x^2 - 1, & \text{if } x = 1 \\
  x^2 + 1, & \text{if } x < 1 
\end{cases} \]

(1) Is \( f(x) \) well defined at \( x = 1 \)? Find \( f(1) \).

(2) Compute the values of \( f \) at the following points approaching 1 using calculator. Then guess the left limit.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>0.9</th>
<th>0.95</th>
<th>0.99</th>
<th>0.999</th>
<th>left limit as ( x \to 1^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(3) Compute the values of \( f \) at the following points approaching 1 using calculator. Then guess the right limit.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.5</th>
<th>1.1</th>
<th>1.05</th>
<th>1.01</th>
<th>1.001</th>
<th>right limit as ( x \to 1^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(4) Does the two-sided limit of \( f(x) \) as \( x \) approaches 1 exist? What is the limit?

(5) Is \( f(x) \) continuous at \( x = 1 \)? Why?

2. The following is the graph of a piecewise function.
Try to fill out the table below and discuss the limits at the integer points.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x) (may not be defined)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left limit (may be infinity)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right limit (may be infinity)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limit (may not exist)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Compute the following the limits:
   (1) \( \lim_{x \to 2} (x^3 - 3x^2 + 2) \);  
   (2) \( \lim_{t \to 0} \sqrt{t^2 + 1} \);  
   (3) \( \lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 1} \);  
   (4) \( \lim_{u \to 1} \frac{u^2 + 1}{u + 1} \);  
   (5) \( \lim_{x \to 3} \frac{x + 2}{x - 3} \).

4. Compute the following fractional limits:
   (1) \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \);  
   (2) \( \lim_{t \to 1} \frac{t^2 - 4t - 5}{t^2 + 5t + 4} \);  
   (3) \( \lim_{h \to 0} \frac{(1 + h)^2 - 1}{h} \).

5. Suppose \( f(x) = x^2 \).
   (1) Compute \( f(1) \) and \( f(1 + h) \) for some parameter \( h \);
   (2) Treat \( h \) as a variable and evaluate the limit as \( h \) approaches 0:  
       \( \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} \).

(Hard) 6. Suppose \( f(x) = x^2 - 3x + 4 \).
   (1) Compute \( f(x + h) \) for some parameter \( h \).
   (2) Treat \( h \) as a variable and \( x \) as a constant. Evaluate the limit as \( h \) approaches 0:  
       \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \). Your answer should be an expression involving \( x \).

**Answer to Homework 2**

1. (1) Yes. \( f(1) = 4 \).

   (2)
   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   x & 0.5 & 0.9 & 0.95 & 0.99 & 0.999 \\
   \hline
   f(x) & 2.333 & 2.052 & 2.026 & 2.005 & 2.0005 \\
   \hline
   \end{array}
   \]

   (3)
   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   x & 1.5 & 1.1 & 1.05 & 1.01 & 1.001 \\
   \hline
   f(x) & 1.8 & 1.952 & 1.976 & 1.995 & 1.9995 \\
   \hline
   \end{array}
   \]

   (4) The limit exists and equals 2.
   (5) \( f \) is not continuous at \( x = 1 \) because the limit doesn't equal the functional value \( f(1) = 4 \).
2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$ (may not be defined)</td>
<td>2</td>
<td>Un-def</td>
<td>0</td>
<td>2</td>
<td>0.5</td>
<td>2</td>
<td>Un-def</td>
</tr>
<tr>
<td>Left limit (may be infinity)</td>
<td>2</td>
<td>1</td>
<td>$-\infty$</td>
<td>2</td>
<td>$-1$</td>
<td>$+\infty$</td>
<td>0</td>
</tr>
<tr>
<td>Right limit (may be infinity)</td>
<td>1</td>
<td>$+\infty$</td>
<td>$-\infty$</td>
<td>2</td>
<td>$-1$</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Limit (may not exist)</td>
<td>DNE</td>
<td>DNE</td>
<td>DNE</td>
<td>2</td>
<td>$-1$</td>
<td>DNE</td>
<td>0</td>
</tr>
<tr>
<td>Continuity</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

3. (1) $-2$; (2) 1; (3) 0; (4) 1; (5) DNE.

4. (1) 4; (2) $-2$; (3) 2.

5. (1) $f(1) = 1$ and $f(1 + h) = (1 + h)^2 = 1 + 2h + h^2$.

   (2) The limit is 2. (Same as problem 4 (3).)

6. (1) $f(x + h) = (x + h)^2 - 3(x + h) + 4 = x^2 + 2xh + h^2 - 3x - 3h + 4$.

   (2) The limit is $2x - 3$. 