Problems:

1. Suppose you deposit 1 dollar into some bank account and the account annual compound interest rate is set to be 100%. Assume the interest is paid several times in a year with equal time segments. Use your calculator to compute the total balance after a year if the interest is paid in the following ways:
   (1) every six months (a half year);  (2) every month;  (3) every day ($\frac{1}{365}$ year);
   (4) every hour ($\frac{1}{365 \cdot 24}$ year);  (5) every minute ($\frac{1}{365 \cdot 24 \cdot 60}$ year).
   Then do the following:
   (6) In general, if the interest is paid $n$ times a year, over equal time segment, write down a general formula for your total balance after a year.
   (7) Check how the total balance changes as $n$ increases in part (1) through (5).
   (8) Given that the decimal expansion of the natural constant $e \approx 2.71828182845905$, check if the total balance in part (1) through (5) approaches $e$ as $n$ gets larger.

2. Suppose you deposit $10000 into some savings account. The annual compound interest rate is 5%.
   (1) Assume the interest is paid every three months. Write the total balance after $x$ years as a function of $x$.
   (2) Use your calculator to compute the total balance after 3 years.
   (3) Assume the interest is paid continuously compounded. Write the total balance after $x$ years as a function of $x$.
   (4) Use your calculator to compute the total balance after 3 years.

3. In general, assume you deposit $A$ dollars into some savings account. The annual compound interest rate is $r$.
   (1) Assume the interest is paid $n$ times a year. Write the total balance after $x$ years as a function of $x$.
   (2) Assume the interest is paid continuously compounded. Write the total balance after $x$ years as a function of $x$.

4. Simplify the following expressions of exponents.
   (1) $\sqrt{\frac{e^{2x}}{e^{3x+1}}}$;  (2) $\left(\frac{e^{4x-5} \cdot e^{-2x}}{e^{3x+1} \cdot e^{2}}\right)^{2}$;  (3) $\frac{e^{0.5x} \cdot (e^{4x})^{4}}{\sqrt{e^{3x}}}$.

5. Which of the following statements about $y = e^x$ is false?
   (A) This function is always positive for all real numbers $x$;
   (B) This function passes through the point (0, 1);
   (C) This function approaches the $x$-axis when $x$ is a very large positive number;
   (D) This function is all the way increasing.

6. Compute the following values using the definition of logarithmic functions without the help of calculator.
(1) \( \log_2 8 \); (2) \( \log_3 9 \); (3) \( \log_5 \frac{1}{25} \); (4) \( \log_3 1 \); (5) \( \log_4 \frac{1}{4} \); (6) \( \ln 1 \); (7) \( \ln e^2 \); (8) \( \ln \frac{1}{e} \).

7. Compute the following values using the fact that exponential functions and logarithmic functions are inverse functions:

(1) \( \log_2 (2^{10}) \); (2) \( \log_3 \left( \frac{1}{3} \right) \); (3) \( \ln(e^4) \); (4) \( e^{\ln 15} \); (5) \( 3^{\log_3 16} \);

(6) \( \log_4 \left( \frac{1}{4} \right)^{3x} \); (7) \( 5^{\log_5 (x^2+3)} \); (8) \( \ln(e^{5x}) \); (9) \( e^{\ln(x+1)} \); (10) \( e^{2\ln(x+3)} \).

8. Simplify the following expressions of logarithmic functions:

(1) \( \log_2 x + \log_2 8 \); (2) \( \log_2 \left( \frac{1}{4} \right) \); (3) \( \log_3 (x^4) - \log_3 \left( \frac{1}{x+1} \right) + \log_3 (9^x) \); (4) \( \ln x + \ln(x^3) + \ln(x^3) \); (5) \( \ln \frac{1}{x} - \ln \frac{1}{x^2} \).

9. Solve the following equations for the values of \( x \).

(1) \( e^x = 2 \); (2) \( \ln x = \frac{1}{2} \); (3) \( 3e^{4x} = 6 \); (4) \( 4 \ln x^2 = 8 \); (5) \( e^{2x} = e^{x^2-2x} \).

10. (1) Suppose we know a relation between two variables \( x \) and \( y \) is \( \ln(10y + 20) = x^2 + 1 \). Can you isolate \( y \) and write \( y \) as a function of \( x \)?

(2) Suppose we know a relation between two variables \( x \) and \( y \) is \( e^{4y + 2} - x^2 = \ln x \). Can you isolate \( y \) and write \( y \) as a function of \( x \)?

11. Which of the following statements about \( \ln x \) is false?

(A) \( \ln x \) is defined only for positive numbers \( x \);
(B) \( \ln x \) is all the way increasing;
(C) \( \ln x \) is always positive;
(D) \( \ln x \) approaches the negative \( y \)-axis as \( x \) is very close to 0;
(E) \( \ln x \) passes through the point \( (1, 0) \).

12. All the following calculation statements are incorrect! Check each one of them until you know where the mistake is. This problem is important because in practice you are very likely to make mistakes like these. Doing this problem will help you clarify all the simplification rules of exponential functions and logarithmic functions.

(1) \( (\ln 2) \cdot (\ln 3) = \ln 6 \);
(2) \( e^{2x} \cdot e^4 = e^{8x} \);
(3) \( \frac{\ln x}{\ln(x+1)} = \frac{x}{x+1} \);
(4) \( \ln(x^2) = (\ln x)^2 \);
(5) \( \ln(x^2 + y^2) = \ln(x^2) + \ln(y^2) = 2 \ln x + 2 \ln y \);
(6) \( \ln \sqrt{x^2+1} = \ln(x^2 + 1) \);
(7) \( e^{e^x} = e^x \);
(8) \( \frac{1}{\ln x} = \ln(x^{-1}) = \ln \frac{1}{x} \);
(9) \( \ln(3e^x) = 3x \);
(10) \( e^{2\ln x} = 2x \).
1. (1) \((1 + \frac{1}{2})^2 = 2.25\); (2) \((1 + \frac{1}{12})^{12} = 2.613\); (2) \((1 + \frac{1}{365})^{365} = 2.7145\); (4) \((1 + \frac{1}{8760})^{8760} = 2.71812\); (5) \((1 + \frac{1}{525600})^{525600} = 2.7182792\). (6) \((1 + \frac{1}{n})^n\); (7) As \(n\) increases, the total balance increases as well; (8) The total balance in part (1) to (5) is approaching \(e\).

2. (1) \(10000(1 + \frac{5\%}{4})^{4}x\); (2) \(11607.5\); (3) \(10000 \cdot e^{0.05x}\); (4) \(11618.3\).

3. (1) \(A(1 + r)^{nx}\); (2) \(Ae^{rx}\).

4. (1) \(e^{-x} - \frac{1}{2}\); (2) \((e^{-x} - 8)^{2} = e^{-2x} - 16\); (3) \(e^{16x}\).

5. (C) \(e^x\) is all the way increasing. The approaching line will occur when \(x\) is a large negative number. I.e. when \(x \to -\infty\).

6. (1) 3; (2) 2; (3) -2; (4) 0; (5) -1; (6) 0; (7) 2; (8) -1.

7. (1) 10; (2) -7; (3) 4; (4) 15; (5) 16; (6) -3x; (7) \(x^2 + 3\); (8) 5x; (9) \(x^4 + 1\); (10) \((x + 3)^2\).

8. (1) \(\log_2 8x\); (2) \(\log_2(2^{-2x}) = -2x\); (3) \(\log_3(x^4(x^2 + 1)) + 2x\); (4) \(\ln x^6 = 6\ln x\); (5) \(-\ln x - (-2)\ln x = \ln x\).

9. (1) \(x = \ln 2\); (2) \(x = e^{0.5}\); (3) \(x = \frac{1}{4}\ln 2\); (4) \(x = e\); (5) \(2x = x^2 - 2x\), then \(x = 0\) or \(x = 4\).

10. (1) \(y = \frac{1}{10}(e^{x^2 + 1} - 20)\); (2) \(e^{4y+2} = x^2 + \ln x\), then \(4y + 2 = \ln(x^2 + \ln x)\) and \(y = \frac{1}{2}(\ln(x^2 + \ln x) - 2)\).

11. (C) \(\ln x\) can have positive, negative or zero values.

12. (1) No formula for the LHS; (2) LHS should equal \(e^{2x+4}\); (3) no formula for the LHS; (4) LHS should equal \(2\ln x\); (5) no formula for the LHS; (6) LHS should equal \(\frac{1}{2}\ln(x^2 + 1)\); (7) no formula for the LHS, the RHS should equal \(e^{0.5x}\); (8) no formula for the LHS; (9) LHS should equal \(x^3\); (10) LHS should equal \(x^2\).