Topics in this homework:

Topic 1 Approximation using midpoint rule
1. Method of approximation using Riemann sum at midpoints.
2. Approximating behavior as \( n \to \infty \), where \( n \) is the number of subintervals.

Topic 2 Improper integrals
1. Definition of improper integrals. Convergence and divergence of improper integrals.
2. Evaluation of improper integrals.

Problems:

1. True and False questions:
   (1) Riemann sum is used to approximate an indefinite integral.
   (2) To approximate integral \( \int_0^2 (4x^2 + 1)\,dx \) using 4 subintervals in the midpoint rule, the midpoints we need to pick are 0.25, 0.75, 1.25, 1.75.
   (3) The functional value of \( f(x) = 4x^2 + 1 \) at midpoint 0.25 in the first subinterval \([0, 0.5]\) is 1.25. So the corresponding approximating column area is 1.25.
   (4) \( \frac{1}{x^3} \) approaches 0 when \( x \) goes to \( +\infty \) or \( -\infty \).
   (5) \( e^{2x} \) approaches 0 when \( x \) goes to \( +\infty \) and approaches \( +\infty \) when \( x \) goes to \( -\infty \).
   (6) \( e^{-2x} \) approaches 0 when \( x \) goes to \( +\infty \) and approaches \( +\infty \) when \( x \) goes to \( -\infty \).
   (7) An improper integral is said to divergent if its value is infinite.

2. (1) Fill in the table below to complete the midpoint rule to approximate integral \( \int_1^5 (x + \frac{1}{2})^2\,dx \) using 4 subintervals. The first row is already given:

<table>
<thead>
<tr>
<th>Subinterval</th>
<th>Width of subinterval</th>
<th>Midpoint</th>
<th>Function value at Midpoint</th>
<th>Column area</th>
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<tbody>
<tr>
<td>[1, 2]</td>
<td>1</td>
<td>1.5</td>
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Total area

(2) Compute this integral \( \int_1^5 (x + \frac{1}{2})^2\,dx \) to find its actual value.
3. Fill in the table below to complete the midpoint rule to approximate definite integral \( \int_0^2 e^{3x} \, dx \) using 5 subintervals (use 2 decimal places for numbers.)

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<tr>
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<th>Function value at Midpoint</th>
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Total area

(2) Compute this integral \( \int_0^2 e^{3x} \, dx \) to find its actual value.

4. (1) Find the antiderivative of function \( f(x) = \frac{4}{x^3} \);
(2) Evaluate definite integral \( \int_1^{\infty} \frac{4}{x^3} \, dx \);
(3) Evaluate improper integral \( \int_1^{\infty} \frac{4}{x^3} \, dx \);
(4) Evaluate improper integral \( \int_{-\infty}^{-2} \frac{4}{x^3} \, dx \).

5. (1) Find the antiderivative of function \( f(x) = \frac{1}{\sqrt{4-x}} \) (using substitution);
(2) Evaluate the definite integral \( \int_0^3 \frac{1}{\sqrt{4-x}} \, dx \);
(3) Evaluate the improper integral \( \int_{-\infty}^{3} \frac{1}{\sqrt{4-x}} \, dx \).

6. Evaluate the following improper integrals. (Note. Some of them may be divergent.)
(1) \( \int_0^{+\infty} xe^{-x^2} \, dx \); (2) \( \int_0^{+\infty} 6e^{-3x} \, dx \); (3) \( \int_{-\infty}^{0} 6e^{-3x} \, dx \); (4) \( \int_0^{\infty} \frac{x^2}{\sqrt{x^3 + 1}} \, dx \).

Homework 4 Solution

1. (1) F; (2) T; (3) F; (4) T; (5) F; (6) T; (7) T.

2. (1) Approximation is 54; (2) Actual value of this integral is 54.333.

3. (1) Approximation is 126.42; (2) Actual value of this integral is 134.14.

4. (1) \(-2x^{-2} + C\); (2) \( \frac{2}{3} \); (3) 2; (4) \(-\frac{1}{2}\).

5. (1) \(-2\sqrt{4-x} + C\); (2) 2; (3) Divergent.

6. (1) \( \frac{1}{2} \); (2) \( 2e \); (3) Divergent; (4) Divergent.