Topics in this homework:

Topic 1 Integration by parts
1. Formula of integration by parts: \( \int f(x)g(x)dx = f(x)G(x) - \int f'(x)G(x)dx \).
2. Evaluate integrals using a combination of substitution and integration by parts.

Problems:

1. (1) Find an antiderivative of \( e^{5x} \) using substitution \( u = 5x \).
   (2) Evaluate the integral \( \int xe^{5x}dx \) using integration by parts and part (1), by identifying functions \( f(x) = x \) and \( g(x) = e^{5x} \).

2. (1) Find an antiderivative of \( \sin(2x) \) using substitution \( u = 2x \).
   (2) Evaluate the integral \( \int x\sin(2x)dx \) using integration by parts and part (1), by identifying functions \( f(x) = x \) and \( g(x) = \sin 2x \).

3. (1) Find an antiderivative of \( e^{2x} \) using substitution.
   (2) Evaluate the integral \( \int e^{2x}(1-3x)dx \) using integration by parts and part (1).

4. (1) Find an antiderivative of \( (2x+3)^3 \) using substitution.
   (2) Evaluate the integral \( \int x(2x+3)^3dx \) using integration by parts and part (1).

5. Use integration by parts to evaluate the following integrals. Note the method of substitution may be needed as an intermediate step.
   (1) \( \int x^5\ln xdx \); (2) \( \int x\sqrt{2x+1}dx \); (3) \( \int \frac{x+3}{e^{2x}}dx \);

6. (1) Verify the result \( \int x\cos xdx = x\sin x + \cos x + C \) using integration by parts;
   (2) Use (1) to evaluate \( \int x^2\sin xdx \) using integration by parts;
   (3) Use (2) to find \( \int_0^\pi x^2\sin xdx \).
7. The following is the graph of function $y = \sqrt{1-x}$ (top curve) and $y = x\sqrt{1-x}$ (bottom curve) from 0 to 1. Find the area of the shaded region between these two curves.

Homework 3 Solution

1. (1) $\frac{1}{5}e^{5x}$; (2) $\frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$.

2. (1) $-\frac{1}{2}\cos 2x$; (2) $\frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$.

3. (1) $\frac{1}{2}e^{2x}$; (2) $\frac{1}{2}(1 - 3x)e^{2x} + \frac{3}{4}e^{2x} + C$.

4. (1) $\frac{1}{8}(2x + 3)^4$; (2) $\frac{1}{8}x(2x + 3)^4 - \frac{1}{80}(2x + 3)^5 + C$.

5. (1) $\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$; (2) $\frac{1}{3}x(2x + 1)^{3/2} - \frac{1}{15}(2x + 1)^{5/2} + C$; (3) $-\frac{1}{2}(x + 3)e^{-2x} - \frac{1}{4}e^{-2x} + C$;

6. (1) Set $f(x) = x$ and $g(x) = \cos x$. Apply integration by parts.
(2) Set $f(x) = x^2$ and $g(x) = \sin x$. Apply integration by parts:
\[
\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C
\]
(3) $\int_0^\pi x^2 \sin x \, dx = \pi^2 - 4$.

7. Write the area as a definite integral $\int_0^1 (\text{top curve} - \text{bottom curve}) \, dx = \int_0^1 (\sqrt{1-x} - x\sqrt{1-x}) \, dx$. The value of this definite integral is $\frac{2}{5}$. 

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