1. For the following functions, identify how many independent variables are there in each function:
   (1) $z = x^2 + y^2$; (2) $f(x) = e^x + x^2$; (3) $V = x^2 + y^2 + z^2$.

2. Given a function $f(x, y) = x^2 + 2xy + y^2$, find the following values;
   (1) $f(1, 2)$; (2) $f(-2, 3)$; (3) $f(0, 0)$.

3. Given a function $w = xyz + x + y^2 + z^3$, find the following values;
   (1) $f(3, 2, 1)$; (2) $f(0, 0, 0)$; $f(-3, 0, -2)$.

4. Given an arbitrary rectangular box with dimensions $x, y, z$,
   (1) Find the volume function $V = f(x, y, z)$ in terms of the dimensions $x, y, z$.
   (2) If we want to construct an open box (i.e. without lid) of dimensions $x, y, z$ as in the picture, determine the function of the total surface area in terms of $x, y, z$.
   (3) Assume $x = 2, y = 3, z = 4$. Find the volume and the surface area of this open box.

5. The graph on the right is the graph of surface defined by $z = 9 - x^2 - y^2$ for positive $x$ and $y$.
   (1) Find the trace curve function when $x = 1$ and $x = 2$ respectively.
   (2) Find the trace curve function when $y = 1$ and $y = 2$ respectively.
   (3) Match the above four curve functions with the four specific curves indicated in the graphs below:
6. This problem will compute the two partial derivatives of the function $z = f(x,y) = x^2y + xy^2 + 3x + 2y + 5$ in steps:

(1) When we take $y = 7$, the function $z$ is $z = f(x) = 7x^2 + 7^2x + 3x + 2 \cdot 7 + 5$. Find $\frac{dz}{dx}$.
(2) When we take $y = 13$, the function $z$ is $z = f(x) = 13x^2 + 13^2x + 3x + 2 \cdot 13 + 5$. Find $\frac{dz}{dx}$.
(3) Now use $z = f(x, y) = yx^2 + y^2x + 3x + 2 \cdot y + 5$. Find the partial derivative $\frac{\partial z}{\partial x}$ by treating $y$ as a constant.
(4) When we take $y = 7$, the function $z$ is $z = f(y) = 7^2y + 7y^2 + 3 \cdot 7 + 2y + 5$. Find $\frac{dz}{dy}$.
(5) When we take $y = 13$, the function $z$ is $z = f(y) = 13^2y + 13y^2 + 3 \cdot 13 + 2y + 5$. Find $\frac{dz}{dy}$.
(6) Now use $z = f(x, y) = x^2y + xy^2 + 3x + 2y + 5$. Find the partial derivative $\frac{\partial z}{\partial y}$ by treating $x$ as a constant.

7. Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following functions:
(1) $z = f(x, y) = 2x + 3y + 4$; (2) $z = f(x, y) = x^3y^2$;

Solution to Homework 11

1. (1) 2 variables; (2) 1 variable; (3) 3 variables.
2. (1) 9; (2) 1; (3) 0.
3. (1) 14; (2) 0; (3) -11.
4. (1) $V = xyz$; (2) $S = 2xz + 2yz + xy$; (3) $V = 24$ and $S = 46$.
5. (1) when $x = 1$, $f(y) = 9 - 1^2 - y^2 = 8 - y^2$; when $x = 2$, $f(y) = 9 - 2^2 - y^2 = 5 - y^2$;
(2) when $y = 1$, $f(x) = 9 - y^2 - 1^2 = 8 - x^2$; when $y = 2$, $f(x) = 9 - x^2 - 2^2 = 5 - x^2$.
(3) $x = 1 \leftrightarrow$ curve 1; $x = 2 \leftrightarrow$ curve 2; $y = 1 \leftrightarrow$ curve 3; $y = 2 \leftrightarrow$ curve 4.
6. (1) $7 \cdot 2x + 7^2 \cdot 1 + 3 + 0 + 0 = 14x + 52$; (2) $13 \cdot 2x + 13^2 \cdot 1 + 0 + 0 = 26x + 172$;
(3) $y \cdot 2x + y^2 \cdot 1 + 3 + 0 + 0 = 2xy + y^2 + 3$;
(4) $7^2 \cdot 1 + 7 \cdot 2y + 0 + 2 + 0 = 14y + 51$; (5) $13^2 \cdot 1 + 13 \cdot 2y + 0 + 2 + 0 = 26y + 171$;
(6) $x^2 \cdot 1 + x + 2x + 0 + 2 + 0 = x^2 + 2xy + 2$.
7. (1) \( \frac{dz}{dx} = 2 \); \( \frac{dz}{dy} = 3 \); (2) \( \frac{dz}{dx} = 3x^2y^2 \); \( \frac{dz}{dy} = 2x^2y \).