CONCEPTUAL MATHEMATICS, a review by Scott W. Williams

Not long ago, I spoke with a professor at strong HBCU department. Her Ph.D. was nearly twenty years ago, but I shocked her with the following statement, "Most of our beginning graduate students [even those in Applied Mathematics] are entering with the basic knowledge and language of Category Theory. These days one might find Chemists, Computer Scientists, Engineers, Linguists and Physicists expressing concepts and asking questions in the language of Category Theory because it slices across the artificial boundaries dividing algebra, arithmetic, calculus, geometry, logic, topology. If you have students you wish to introduce to the subject, I suggest a delightfully elementary book called Conceptual Mathematics by F. William Lawvere and Stephen H. Schanuel" [Cambridge University Press 1997, $35 on Amazon.com] (ISBN: 0521478170 | ISBN-13:9780521478175).

From the introduction: "Our goal in this book is to explore the consequences of a new and fundamental insight about the nature of mathematics which has led to better methods for understanding and usual mathematical concepts. While the insight and methods are simple ... they will require some effort to master, but you will be rewarded with a clarity of understanding that will be helpful in unraveling the mathematical aspect of any subject matter."

Who are the authors? Lawvere is one of the greatest visionaries of mathematics in the last half of the twentieth century. He characteristically digs down beneath the foundations of a concept in order to simplify its understanding. Though Schanuel has published research in diverse areas of Algebra, Topology, and Number Theory, he is known as a great teacher. The book is an edited transcript of a course taught by Lawvere and Schanuel to American undergraduate math students. The book was actually chosen as one of the items in the Library of Science Book Club. The concepts of Category Theory in Conceptual Mathematics are presented in the same way Lawvere and Schanuel implemented it, in a real classroom setting, addressing common questions of students (yes these are real people) at crucial points in the book.

The book comes with thirty-three Sessions instead of Chapters. Some Sessions can be understood in a single class or hour. Others may take longer. There are also numerous Examples, Problems, and five Tests of the student's understanding.

The title of Session 1 is "Galileo and the flight of a bird" and motivates the notion map. The sixth part of Session 5 is called "Stacking in a Chinese restaurant" and helps motivate sections and retractions. Session 10 motivates the Brouwer Fixed Point Theorem. Less you think this is all Abstract Mathematical nonsense, Session 15 is called "Objectification of properties in dynamical systems." The title of Session 20 is "Points of an object."
I have recommended Lawvere and Schanuel to motivated high school students. I certainly suggest this clearly written "Conceptual Mathematics" for undergraduates. I even suggest it for the mathematician who needs a refresher on modern concepts.