Multiple Access Channel with State Uncertainty at Transmitters

Shaofeng Zou
Dep. of EECS
Syracuse University
Syracuse, NY 13244 USA
Email: szou02@syr.edu

Yingbin Liang
Dep. of EECS
Syracuse University
Syracuse, NY 13244 USA
Email: yliang06@syr.edu

Shlomo Shamai (Shitz)
Department of EE
Technion-Israel Institute of Technology
Technion City, Haifa 32000 Israel
Email: sshlomo@ee.technion.ac.il

***Abstract*—Two-user fading multiple access channel (MAC) is investigated, which is corrupted by random fading coefficients and additive Gaussian noise. It is assumed that the channel is block fading, and each transmitter knows only its own channel state to the receiver, but does not know the other transmitter's channel state. The receiver has full knowledge of channel state information (CSI). The performance measure, the expected capacity region over channel statistics, is studied for two scenarios. For the first scenario, in which user 1 has multiple states, and user 2 has one state, most part of the boundary of the expected capacity region is characterized. Interestingly, these rate points are also on the boundary of the capacity region (i.e., the best achievable rate pairs) when the CSI is fully known at both transmitters. Furthermore the expected capacity region is fully characterized for some asymptotic regimes. For the second scenario, in which both users 1 and 2 have two states, a number of achievable regions are studied, and are demonstrated to be close to an outer bound numerically.**

I. Introduction

Channels with state uncertainty at transmitters model many typical wireless communication scenarios, in which it is difficult to feed back the channel state information (CSI) to transmitters from receivers. It is thus desirable to design transmission schemes that do not exploit the CSI but still adapt to the actual channel state to achieve as good performance as the channel supports. A broadcast approach has been proposed in [1] for the single-user fading channel, in which the transmitter has no access to the CSI. The basic idea is to split the entire message into multiple layers and encode them by superposition coding. The receiver decodes up to a certain layer depending on the actual channel state. In this way, although the transmitter does not know the CSI, the transmission rate still adapts to the channel realization. This approach has been subsequently generalized to study various channel models, e.g., the fading multiple-input multiple-output (MIMO) channel in [2], [3], the fading multiple access channel (MAC) in [4], and the fading wiretap channel in [5].

In particular, the fading MAC with the CSI not known at transmitters was studied in [4], in which the channels from two transmitters to the receiver are corrupted by continuous fading gain coefficients and additive Gaussian noise. A broadcast approach was designed, and an expected achievable rate region over channel statistics was derived. A two-user MAC model with each user having two states (i.e., weak and strong states) was studied in [6]. Here, the focus was on the sum rate corresponding to weak states versus the sum rate corresponding to strong states. The capacity region that contains all such achievable sum rate pairs was characterized, and was shown to be achieved by the time-sharing scheme. A random access channel was further studied in [6] and [7], in which each transmitter has states to be either on or off. The expected throughput was derived and was shown to be within one bit to the developed upper bound. A MAC model with transmitters synchronously transmitting packets in a bursty manner was studied in [8].

In this paper, we study the two-user MAC, in which each transmitter's channel to the receiver is corrupted by a random fading gain coefficient that may take a number of states and additive Gaussian noise. Differently from [4], [6], [7] that assume no CSI at any transmitter, we assume that the channel state is known to its corresponding transmitter, but not known to the other transmitter. This occurs often in the uplink cellular networks that each individual user estimates its own state via the base station's feedback. Such information is usually not provided to or shared with other users. The channel is assumed to be block fading, and it is required that each message need to be transmitted within one block, and hence coding over blocks is not allowed. Similar to [4], the performance measure of interest is the pair of the two users' expected rates, and our goal is to study the expected capacity region that contains all such pairs of expected rates. Such expected capacity captures the average throughput of the system over a long range, but with restrictive delay constraint on individual messages. We note that such rate pairs can be viewed as achievable variable-to-fixed rate pairs in terms of the notion introduced in [9]. We also note that this performance measure focuses on individual user's expected rate, and is different from that studied in [6], [7], which focused on the sum rate of users corresponding to each individual state.

In order to design achievable schemes, our major challenge lies in the channel state uncertainty at transmitters. Although each transmitter knows its own channel state, it does not know the other transmitter's channel state to efficiently adapt its scheme. A natural scheme is to apply the broadcast approach, which, however, is not optimal to achieve the capacity in general. In this paper, we focus on designing schemes that are optimal for certain regimes. In particular, we study two scenarios. For the first scenario, in which transmitter 1 has multiple states and transmitter 2 has one state, the challenge lies in the state uncertainty at transmitter 2. We show that a superposition encoding and sequential decoding scheme achieves most part of the boundary of the expected capacity region. Somewhat interestingly, these rate points on the boundary are the best achievable rate pairs even with the full CSI at both transmitters. Furthermore, we characterize the full capacity region for some asymptotic regimes. We then study the second scenario, in which both transmitters have two states. We apply our scheme for scenario 1 and obtain the corresponding achievable region. This region is shown to be larger than that based on time-sharing scheme and is close to an outer bound numerically.

This paper is organized as follows. In Section II, we introduce our system model. In Sections III and IV, we present our results for scenario 1 corresponding to transmitter 1 having two states and multiple states, respectively. In Section V, we present our results for scenario 2. Finally, in Section VI we conclude our paper with a few remarks.

II. Channel Model

We study the fading MAC, in which two users transmit to one receiver. The channel input-output relationship for one channel use is given by

$$Y = h_1 X_1 + h_2 X_2 + Z \tag{1}$$

where X_1 and X_2 denote the channel inputs, Y denotes the channel output, and Z denotes the Gaussian noise variable with mean zero and variance N. We assume that each fading gain coefficient h_j (for $j = 1, 2$) can take K realizations (i.e., states), i.e., $0 < \alpha_{j1} < \alpha_{j2} < ... < \alpha_{jK} = 1$, and $P(h_j = \alpha_{ji}) = q_{ji}$ with $\sum_{i=1}^{K} q_{ji} = 1$. We adopt the block fading model, i.e., the fading coefficients remain the same during each block and change independently from one block to another. It is assumed that the realization of each fading coefficient is known at the corresponding transmitter, but not known at the other transmitter. The receiver is assumed to know both fading coefficients. It is also assumed that the two transmitters are subject to individual power constraints P_1 and P_2, respectively, and the power of each user cannot be allocated among different states. As we comment later, our results can be naturally extended to the case with power adaption across states.

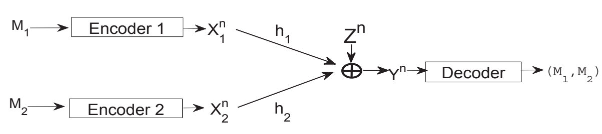

Fig. 1. The system model for the two-user fading MAC

In this paper, we require each message to be transmitted within one block, and hence coding over blocks is not allowed. We are interested in the performance of the expected capacity region over a large number of blocks. More precisely, suppose the rate pair $(R_1(h_1, h_2), R_2(h_1, h_2))$ is achievable as a function of the state pair (h_1, h_2), and then an expected achievable rate pair is given by $(E_{h_1,h_2}[R_1(h_1, h_2)], E_{h_1,h_2}[R_2(h_1, h_2)])$. Then the expected capacity region is defined to contain all expected achievable rate pairs.

In this paper, we study the expected capacity region for the following two scenarios: (1) scenario 1: user 1 has multiple states and user 2 has one state; (2) scenario 2: both users have two states.

III. Scenario 1: An Illustration of a Simple Case

In this section, we study a simple case of scenario 1 to illustrate our results, in which user 1 has two states, and user 2 has one state. We generalize our results to the general case of scenario 1 in Section IV.

For convenience, we assume that $h_2 = 1$, and h_1 takes values α (with probability q_1) and 1 (with probability $1 - q_1$), where $\alpha < 1$. Thus, the channel can be equivalently viewed as a broadcast network with two transmitters and two receivers as follows:

$$Y_1 = \alpha X_1 + X_2 + Z \tag{2}$$
$$Y_2 = X_1 + X_2 + Z \tag{3}$$

We use R_{ji} to denote the achievable rate for such a system, in which j corresponds to transmitter j, and i corresponds to receiver Y_i (i.e., the state i). Then the expected achievable rate pair can be expressed as follows,

$$\begin{aligned} R_1 &= q_1 R_{11} + (1 - q_1) R_{12} \\ R_2 &= q_1 R_{21} + (1 - q_1) R_{22}. \end{aligned} \tag{4}$$

It is clear that for such a system, transmitter 1 has full knowledge of the CSI (because transmitter 2 has only one state), and can hence adopt a scheme that adapts to the channel state. However, transmitter 2 does not know the state of transmitter 1's channel. Since transmitter 1's state affects signal interference to decode transmitter 2's message at the receiver, it is natural for receiver 2 to implement a two-layer broadcast approach to adapt to channel states of transmitter 1. Somewhat interestingly, we show that by exploiting properties of the capacity of the MAC, a single-layer scheme for transmitter 2 (together with an adaptive scheme for transmitter 1) achieves most part of the boundary of the expected capacity region. More importantly, these boundary points are on the boundary of the expected capacity region (i.e., the best achievable rate pairs) even with transmitter 2 having the CSI.

In the following, we first present the expected capacity region when transmitter 2 knows the full CSI (i.e., the channel state of transmitter 1), which serves as an outer bound on the expected capacity region when transmitter 2 does not know the CSI. We then show that most part of the boundary of this region can be achieved when transmitter 2 does not know the CSI.

Proposition 1. *(Outer Bound) Consider scenario 1 with transmitters 1 and 2 respectively having two and one state and with the CSI at the corresponding transmitter. An outer bound* *on the expected capacity region contains rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned} R_1 &\leq q_1 C(\alpha^2 P_1/N) + (1-q_1)C(P_1/N) \\ R_2 &\leq C(P_2/N) \\ R_1 + R_2 &\leq q_1 C((\alpha^2 P_1 + P_2)/N) + (1-q_1)C((P_1+P_2)/N) \end{aligned} \tag{5}$$

where $C(x) = \frac{1}{2}\log(1+x)$.

The above proposition can be shown by taking expected values for any two rate pairs selected from the capacity regions of the MACs in (2) and (3), respectively. The details of the proof is omitted due to the space limitations.

We next provide an achievable scheme and show that this scheme achieves most part of the boundary of the above outer bound.

Proposition 2. *(Inner Bound) Consider scenario 1 with transmitters 1 and 2 respectively having two and one state, and with the CSI at the corresponding transmitter. An expected achievable region contains rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned} & R_1 \leq q_1 C\left(\tfrac{\alpha^2 P_1}{N}\right) + (1-q_1)C\left(\tfrac{P_1}{N}\right) \\ & R_2 \leq C\left(\tfrac{P_2}{N}\right) \\ & R_1 + R_2 \leq q_1 C\left(\tfrac{\alpha^2 P_1 + P_2}{N}\right) + (1-q_1)C\left(\tfrac{(P_1+P_2)}{N}\right) \\ & R_1 + (1-q_1)R_2 \leq q_1 C\left(\tfrac{\alpha^2 P_1}{N}\right) + (1-q_1)C\left(\tfrac{P_1+P_2}{N}\right). \end{aligned} \tag{6}$$

Outline of the Proof: The basic idea of the scheme is that transmitter 2 sends a Gaussian input signal at a fixed rate, say R_2, and then transmitter 1 adapts its scheme in order to achieve a boundary point of the capacity region of the MAC for each state. This is feasible by exploiting the sequential decoding strategy for the MAC. The details are omitted due to the space limitations. ■

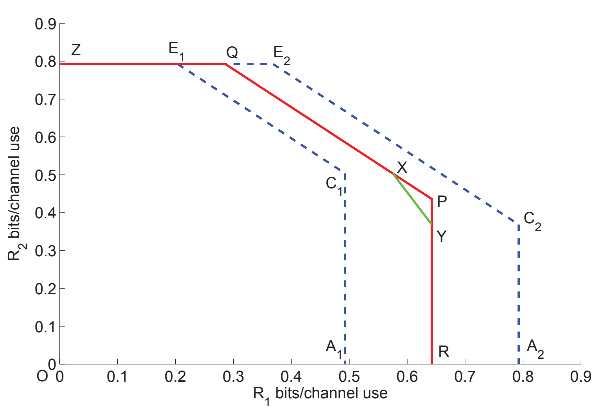

Fig. 2. Inner and outer bounds on the expected capacity region for scenario 1 with users 1 and 2 having two and one state, respectively

In Fig. 2, the pentagons O-Z-E_1-C_1-A_1 and O-Z-E_2-C_2-A_2 denote the capacity regions for the MACs defined in (2) and (3), respectively. We also plot the inner bound (given in Proposition 2) and outer bound (given in Proposition 1) on the expected capacity region respectively as the polygons O-Z-Q-X-Y-R and O-Z-Q-P-R. It can be observed that the inner and outer bounds match in boundary over the lines Z-Q-X and Y-R, which can also be shown analytically by comparing the two bounds given in (6) and (5). Therefore, these lines characterize part of the boundary of the expected capacity region as we state in the following theorem.

Theorem 1. *(Capacity) For scenario 1 with transmitters 1 and 2 respectively having two and one state, and with the CSI at the corresponding transmitter, the lines* Z-Q-X *and* Y-R *in Fig. 2 are on the boundary of the expected capacity region where* Z, Q, X, Y, R *correspond to rate pairs* $(0, C(\frac{P_2}{N}))$, $(qC(\frac{\alpha^2 P_1}{P_2+N}) + (1-q)C(\frac{P_1}{P_2+N}), C(\frac{P_2}{N}))$, $(qC(\frac{\alpha^2 P_1}{P_2+N}) + (1-q)C(\frac{P_1}{P_2+N}) + C(\frac{P_2}{N}) - C(\frac{P_2}{\alpha^2 P_1+N}), C(\frac{P_2}{\alpha^2 P_1+N}))$, $(qC(\frac{\alpha^2 P_1}{N}) + (1-q)C(\frac{P_1}{N}), C(\frac{P_2}{P_1+N}))$, $(qC(\frac{\alpha^2 P_1}{N}) + (1-q)C(\frac{P_1}{N}), 0)$, *respectively. In particular, the line* Q-X *achieves the expected sum capacity.*

Remark 1. *Theorem 1 implies that although transmitter 2 does not know transmitter 1's state, our scheme still achieves most part of the boundary of the capacity region when transmitter 2 has the full CSI.*

Remark 2. *Although our scheme uses a fixed rate* R_2 *for different states of user 1, it achieves most points on the capacity boundary even if* R_2 *can vary for different states of user 1.*

Remark 3. *If power allocation among different states is allowed for users, outer and inner bounds can be derived based on our current results with additional union over all possible power allocation functions. Consequently, the two bounds still match mostly in boundary.*

We next study some asymptotic regimes, in which the inner and outer bounds in Propositions 2 and 1 match asymptotically, and we hence obtain the entire capacity region.

It is clear from Fig. 2 that the only difference between the inner and outer bounds is the gap between X-Y and X-P-Y. We use R_{2X},R_{2Y} to denote the rates of user 2 at point X and Y, respectively. We consider the difference between R_{2X} and R_{2Y}, which is given by

$$D := |R_{2X} - R_{2Y}| = \frac{1}{2}\log\left(1 + \frac{(1-\alpha^2)P_1 P_2}{(N+\alpha^2 P_1)(N+P_1+P_2)}\right).$$

It is clear that if D converges to zero in an asymptotic regime, then the triangle X-P-Y in Fig. 2 reduces to one point, which implies that the inner and outer bounds match asymptotically. Hence, in such a regime, we obtain the entire expected capacity region. We next present two such asymptotic regimes.

Proposition 3. *(Capacity) If both user 1 and user 2 have low power constraints, i.e.,* $P_1 \to 0$, $P_2 \to 0$, *and* $P_1 = aP_2$ *where* a *is a constant, then the inner and outer bounds in Propositions 2 and 1 match each other, thus providing the expected capacity region asymptotically.*

Remark 4. *Although the expected capacity region shrinks to the origin as $P_1 \to 0$ and $P_2 \to 0$, the above result is meaningful because the distance D approaches zero faster than the inner and outer bounds, and hence representing characterization of the capacity region in the asymptotic sense.*

Proposition 4. *(Capacity) If the power of user 1 is large, i.e. $P_1 \to \infty$, and the power P_2 of user 2 is finite, the inner and outer bounds in Propositions 2 and 1 match each other, thus providing the expected capacity region asymptotically.*

IV. Scenario 1: General Case

In this section, we study the general case of scenario 1 with user 1 having multiple states and user 2 having one state. We note that our results can be easily generalized to the case with continuous fading distribution. For notational convenience, we use $\alpha_1, \alpha_2, ..., \alpha_K$ to denote the states that h_1 can take. Without loss of generality, we assume that $0 < \alpha_1 < \alpha_2 < ... < \alpha_K = 1$, and $P(h = \alpha_i) = q_i$ with $\sum_{i=1}^{K} q_i = 1$.

In the following, we generalize our analysis in Section III to obtain inner and outer bounds on the expected capacity region, which match mostly in boundary and hence characterize the most part of the boundary of the expected capacity region. We also provide characterization of the entire expected capacity region in some asymptotic regimes. Since the ideas of proving these results are similar to those in Section III, we omit the details of the proofs and present only the main results.

Proposition 5. *(Outer Bound) Consider scenario 1 with transmitters 1 and 2 respectively having multiple and one state and with the CSI at the corresponding transmitter. An outer bound on the expected capacity region contains rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 &\le \sum_{i=1}^{K} q_i C\left(\frac{\alpha_i^2 P_1}{N}\right), \quad R_2 \le C\left(\frac{P_2}{N}\right) \\ R_1 + R_2 &\le \sum_{i=1}^{K} q_i C\left(\frac{\alpha_i^2 P_1 + P_2}{N}\right). \end{aligned} \tag{7}$$

Proposition 6. *(Inner Bound) Consider scenario 1 with transmitters 1 and 2 respectively having multiple and one state and with the CSI at the corresponding transmitter. An inner bound on the expected capacity region contains rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} &R_1 \le \sum_{i=1}^{K} q_i C\left(\frac{\alpha_i^2 P_1}{N}\right), \quad R_2 \le C\left(\frac{P_2}{N}\right) \\ &R_1 + R_2 \sum_{i}^{K} q_i \le \sum_{j=1}^{i-1} q_i C\left(\frac{\alpha_i^2 P_1}{N}\right) + \sum_{j=i}^{K} q_i C\left(\frac{\alpha_i^2 P_1 + P_2}{N}\right), \\ &\qquad\qquad \textit{for } 1 \le i \le K. \end{aligned} \tag{8}$$

In Fig. 3, each pentagon O-Z-E_i-C_i-A_i denotes the capacity region for the MAC with $h_1 = \alpha_i$ for $i = 1, \ldots, K$. We also plot the boundaries of the inner bound (given in Proposition 6) and outer bound (given in Proposition 5) on the expected capacity region, respectively, as the polygon O-Z-Q-X-Y-R and O-Z-Q-P-R. The inner bound for the general case is different from that in Fig. 2 in that the line X-Y is a polyline

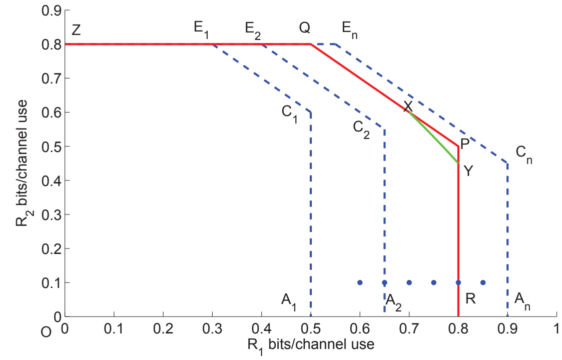

Fig. 3. Inner and outer bounds on the expected capacity region for the general case of scenario 1

instead of a straight line. It can be observed that the two bounds match over the lines Z-Q-X and Y-R, which can also be shown analytically by comparing the two bounds given in (8) and (7). Therefore, these lines characterize part of the boundary of the expected capacity region as we state in the following theorem.

Theorem 2. *(Capacity) The lines Z-Q-X and Y-R in Fig. 3 are on the boundary of the expected capacity region for scenario 1 with transmitters 1 and 2 respectively having multiple and one state and with the CSI at the corresponding transmitter. In particular, the linear line Q-X achieves the sum capacity.*

In order to obtain the expected capacity region in asymptotic regimes, we compare the rates of user 2 at the best state α_n and the worst state α_1. Following the similar arguments to those in Section III, we obtain the following results.

Proposition 7. *(Capacity) If both user 1 and user 2 have low power constraints, i.e., $P_1 \to 0$, $P_2 \to 0$, and $P_1 = aP_2$ where a is a constant, the inner and outer bounds in Propositions 6 and 5 match each other, thus providing the expected capacity region asymptotically.*

We note that Remark 4 is also applicable here.

Proposition 8. *(Capacity) If the power of user 1 is large, i.e. $P_1 \to \infty$, and the power P_2 of user 2 is finite, the inner and outer bounds in Propositions 6 and 5 match each other, thus providing the expected capacity region asymptotically.*

V. Scenario 2: Both Users Have Two States

In this section, we study scenario 2, in which both user 1 and user 2 have two states, and each user knows its own channel state. We assume that both fading coefficients can randomly take one of the two values $\{\alpha, 1\}$ with $0 \le \alpha \le 1$, independently. We let $P(h_j = \alpha) = q_j$ and $P(h_j = 1) = 1 - q_j$ for $j = 1, 2$. Hence the MAC has four states. We can also view the channel equivalently as the following broadcast network with two transmitters and four receivers.

$$\begin{aligned} Y_1 &= \alpha X_1 + \alpha X_2 + Z, \qquad Y_2 = X_1 + \alpha X_2 + Z \\ Y_3 &= \alpha X_1 + X_2 + Z, \qquad Y_4 = X_1 + X_2 + Z \end{aligned} \tag{9}$$

We first provide an outer bound in the following proposition based on taking expectation of the capacity regions of the four MACs with receiver outputs Y_1, Y_2, Y_3 and Y_4.

Proposition 9. *(Outer Bound) Consider scenario 2 with transmitters 1 and 2 both having two states and with the CSI at the corresponding transmitter. An outer bound on the expected capacity region contains rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned} R_1 &\leq q_1 C\left(\frac{\alpha^2 P_1}{N}\right) + (1-q_1) C\left(\frac{P_1}{N}\right) \\ R_2 &\leq q_2 C\left(\frac{\alpha^2 P_2}{N}\right) + (1-q_2) C\left(\frac{P_2}{N}\right) \end{aligned} \tag{10}$$

$$\begin{aligned} R_1 + R_2 \leq\ & q_1 q_2 C\left(\frac{\alpha^2 P_1 + \alpha^2 P_2}{N}\right) \\ & + (1-q_1) q_2 C\left(\frac{P_1 + \alpha^2 P_2}{N}\right) + q_1(1-q_2) C\left(\frac{\alpha^2 P_1 + P_2}{N}\right) \\ & + (1-q_1)(1-q_2) C\left(\frac{P_1 + P_2}{N}\right). \end{aligned}$$

As we show next, an achievable scheme based on superposition and sequential decoding applied for scenario 1 is close but cannot achieve the above outer bound. We describe the scheme as follows. We split the inputs of both user 1 and user 2 into two parts, and the receiver decodes the four parts of inputs sequentially. In particular, the outputs corresponding to four states are given by

$$\begin{aligned} Y_1 &= \alpha\sqrt{1-\beta_1} X_1 + \alpha\sqrt{\beta_1} X_1 + \alpha\sqrt{1-\beta_2} X_2 + \alpha\sqrt{\beta_2} X_2 + Z \\ Y_2 &= \alpha\sqrt{1-\beta_1} X_1 + \alpha\sqrt{\beta_1} X_1 + \sqrt{1-\beta_2'} X_2' + \sqrt{\beta_2'} X_2' + Z \\ Y_3 &= \sqrt{1-\beta_1'} X_1' + \sqrt{\beta_1'} X_1' + \alpha\sqrt{1-\beta_2} X_2 + \alpha\sqrt{\beta_2} X_2 + Z \\ Y_4 &= \sqrt{1-\beta_1'} X_1' + \sqrt{\beta_1'} X_1' + \sqrt{1-\beta_2'} X_2' + \sqrt{\beta_2'} X_2' + Z, \end{aligned} \tag{11}$$

where β_1, β_1', β_2 and $\beta_2' \in [0, 1]$ are the power allocation parameters. We note that since each user knows only its own state, its power allocation adapts across two parts of messages only when its own state changes. The decoding order for each output is to decode part of X_1, part of X_2, remaining part of X_1, and the remaining part of X_2. It can be shown that varying decoding orders and adapting decoding orders based on states do not result in better expected rates. Based on the above scheme, an expected achievable rate region can be obtained.

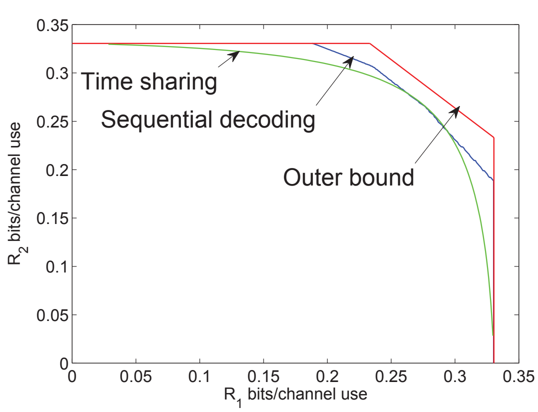

Fig. 4. Comparison of achievable regions and outer bound

Fig. 4 compares the inner bound (labeled as sequential decoding) based on the above scheme and the outer bound given in Proposition 9. It is clear that the two bounds are close, but do not match. It can also be seen from Fig. 4 that the scheme based on sequential decoding outperforms time-sharing for most part of the region.

VI. Conclusion

In this paper, we have studied the expected capacity region of the fading MAC with the CSI known at the corresponding transmitter. For the scenario in which one user has multiple states and the other user has one state, we have characterized most part of the boundary of the expected capacity region. We have also characterized the entire expected capacity region in some asymptotic regimes. For the scenario in which both users have two states, we have derived inner and outer bounds, which are close numerically but do not match each other. This scenario provides an interesting open problem for us to solve in the future, i.e., whether the expected capacity region based on full knowledge of the CSI at transmitters is achievable when the CSI is known only to corresponding transmitters, and if so, what scheme achieves such a region. We will also study the case when the CSI is completely not known at the two users.

Acknowledgment

The work of S. Zou and Y. Liang was supported by a NSF CAREER Award under Grant CCF-10-26565 and by the NSF under Grants CCF-10-26566 and CNS-11-16932. The work of S. Shamai (Shitz) was supported by the Israel Science Foundation (ISF), and the European Commission in the framework of the Network of Excellence in Wireless COMmunications NEWCOM#.

References

[1] S. Shamai, "A broadcast strategy for the Gaussian slowly fading channel," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, June 1997, p. 150.

[2] A. Steiner and S. Shamai, "Multi-layer broadcasting over a block fading MIMO channel," *IEEE Transactions on Wireless Communications*, vol. 6, no. 11, pp. 3937–3945, November 2007.

[3] S. Shamai and A. Steiner, "A broadcast approach for a single-user slowly fading MIMO channel," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2617–2635, October 2003.

[4] S. Shamai, "A broadcast approach for the multiple-access slow fading channel," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, June 2000, p. 128.

[5] Y. Liang, L. Lai, H. V. Poor, and S. Shamai, "Broadcasting over fading wiretap channels," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, July 2012, pp. 234–238.

[6] P. Minero and D. N. C. Tse, "A broadcast approach to multiple access with random states," in *IEEE International Symposium on Information Theory (ISIT)*, June 2007, pp. 2566–2570.

[7] P. Minero, D. N. C. Tse, and M. Franceschetti, "A broadcast approach to random access," in *IEEE Information Theory Workshop (ITW)*, October 2009, pp. 615–619.

[8] M. Medard, J. Huang, A. Goldsmith, S. Meyn, and T. Coleman, "Capacity of time-slotted ALOHA packetized multiple-access systems over the AWGN channel," *IEEE Transactions on Wireless Communications*, vol. 3, no. 2, pp. 486–499, March 2004.

[9] S. Verdú and S. Shamai, "Variable-rate channel capacity," *IEEE Transactions on Information Theory*, vol. 56, no. 6, pp. 2651–2667, June 2010.