PRRR serial chain manipulator

Project Report
MAE 593

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Introduction:

The PRRR serial chain manipulator is redundant. *matlab fmincon* was used to determine the inverse kinematics of the system for a given end effector position.

\[
X_{EE} = D + L_1 \cos \theta_1 + L_2 \cos \theta_2 + L_3 \cos \theta_3 \\
Y_{EE} = L_1 \sin \theta_1 + L_2 \sin \theta_2 + L_3 \sin \theta_3
\]

We define \( f_1(D, \theta_1, \theta_2, \theta_3) \) and \( f_2(D, \theta_1, \theta_2, \theta_3) \) as below and try to minimize the objective function \( g(D, \theta_1, \theta_2, \theta_3) \) as below:

\[
f_1(D, \theta_1, \theta_2, \theta_3) = D + L_1 \cos \theta_1 + L_2 \cos \theta_2 + L_3 \cos \theta_3 - X_{EE}
\]

\[
f_2(D, \theta_1, \theta_2, \theta_3) = L_1 \sin \theta_1 + L_2 \sin \theta_2 + L_3 \sin \theta_3 - Y_{EE}
\]

\[
g(D, \theta_1, \theta_2, \theta_3) = \sqrt{(f_1(D, \theta_1, \theta_2, \theta_3))^2 + (f_2(D, \theta_1, \theta_2, \theta_3))^2}
\]

We use fmincon wherein we try to

\[
\min(g(D, \theta_1, \theta_2, \theta_3)) \text{ such that } 0 \leq D \leq D_{max}
\]
Forward Kinematics:

\((D, L1, L2, L3) = (0.6, 1.5, 1, 2), (\theta_1, \theta_2, \theta_3) = (60, 90, 120)\)

Please see drop box folder for code.
Inverse Kinematics:

$$(X_{EE}, Y_{EE}) = (-2, -1)$$

Please see drop box folder for code.
Workspace:

Please see drop box folder for code.
Circle Tracing:

Please see drop box folder for code.
Ellipse Tracing:

Please see drop box folder for code.
Control:

The Jacobian $J$ is given by

$$ J = \begin{pmatrix} \frac{df_1}{dD} & \frac{df_1}{d\theta_1} & \frac{df_1}{d\theta_2} & \frac{df_1}{d\theta_3} \\ \frac{df_2}{dD} & \frac{df_2}{d\theta_1} & \frac{df_2}{d\theta_2} & \frac{df_2}{d\theta_3} \end{pmatrix} $$

1) Open Loop Control:

Open loop control was achieved using the *matlab ode45* function.

$$ \text{pinv}J = \text{transpose}(J) \ast \text{inverse}(J \ast \text{transpose}(J)) $$

$$ \dot{\theta}_{\text{openloop}} = \text{pinv}J \ast \begin{pmatrix} \ddot{X} \\ \ddot{Y} \end{pmatrix} $$
Timeline for \((D, \theta_1, \theta_2, \theta_3)\)

Please see drop box folder for code for open loop control.
Timeline for $\{D, \theta_1, \theta_2, \theta_3\}$

Please see drop box folder for code for open loop control.
2) Closed Loop Control (Joint Space):

\[ \dot{\theta}_{\text{total}} = \dot{\theta}_{\text{openloop}} + K \ast \begin{pmatrix} \theta_{1\text{desired}} \\ \theta_{2\text{desired}} \\ \theta_{3\text{desired}} \end{pmatrix} - \begin{pmatrix} D \\ \theta_{1} \\ \theta_{2} \\ \theta_{3} \end{pmatrix} \]

\[ \dot{\theta}_{\text{total}} = \text{pinv}\mathbf{J} \ast (\dot{\mathbf{X}}) + K \ast \begin{pmatrix} \theta_{1\text{desired}} \\ \theta_{2\text{desired}} \\ \theta_{3\text{desired}} \end{pmatrix} - \begin{pmatrix} D \\ \theta_{1} \\ \theta_{2} \\ \theta_{3} \end{pmatrix} \]
Timeline for \((D, \theta_1, \theta_2, \theta_3)\)

Please see drop box folder for code for closed loop control.
Timeline for \( (D, \theta_1, \theta_2, \theta_3) \)

Please see drop box folder for code for closed loop control.
3) Closed Loop Control (Task Space):

\[ \dot{\theta}_{\text{closed}} = \text{pinv}J \left\{ \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} + K \left\{ \begin{pmatrix} X_{\text{desired}} \\ Y_{\text{desired}} \end{pmatrix} - \begin{pmatrix} X \\ Y \end{pmatrix} \right\} \right\} \]
Timeline for $(\mathbf{D}, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.
Timeline for $(\mathcal{D}, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.
Manipulability:

The *Jacobian J* is given by

\[
J = \begin{pmatrix}
\frac{df_1}{dD} & \frac{df_1}{d\theta_1} & \frac{df_1}{d\theta_2} & \frac{df_1}{d\theta_3} \\
\frac{df_2}{dD} & \frac{df_2}{d\theta_1} & \frac{df_2}{d\theta_2} & \frac{df_2}{d\theta_3}
\end{pmatrix}
\]

1) Yoshikawa Measure of Manipulability

\[
YMOM = \sqrt[2]{\det(J \ast \text{transpose}(J))}
\]

Please see drop box folder for code for Yoshikawa measure of manipulability.

2) Isotropy Index Measure of Manipulability

\[
U \ast \text{Sigma} \ast \text{transpose}(V) = \text{SVD}(J)
\]
Please see drop box folder for code for Isotropy Index measure of manipulability.

3) **Manipulability Ellipsoids**
Please see drop box folder for code for manipulability ellipsoids.
Appendix

Forward Kinematics http://www.youtube.com/watch?v=yESlf2PsD6g
Inverse Kinematics http://www.youtube.com/watch?v=BlS7HWrkxHA
Workspace http://www.youtube.com/watch?v=f5f1TCPc5Gs
Circle/Ellipse tracing http://www.youtube.com/watch?v=r2oCkZSAfy8
Open Loop Control http://www.youtube.com/watch?v=yIJCD_EZvU
Task Space Closed Loop Control http://www.youtube.com/watch?v=gfCA28yx8k
Joint Space Closed Loop Control http://www.youtube.com/watch?v=OBQs7XfaEeA
Isotropy Index Measure of Manipulability http://www.youtube.com/watch?v=tE7zzhBjgyo
Yoshikawa Measure of Manipulability http://www.youtube.com/watch?v=CpM4n1pHX8w
Manipulability Ellipsoids http://www.youtube.com/watch?v=q7a88xQ6teQ