MTH 538 Review for Exam 2

Problem 1

Calculate the local truncation error for Strang-splitting for the ODE system,

$$\mathbf{u}_t = A\mathbf{u} + B\mathbf{u},$$

where,

(a) with the following matrices,

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

(b) repeat your calculation but with the matrices,

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Problem 2

Compute discrete Fourier transform (DFT) of the vectors,

$$\mathbf{u}^T = (0, 1, 0, -1),$$
$$\mathbf{v}^T = (1, 0, -1, 0),$$
$$\mathbf{w}^T = (1, i, -1, -i)$$

Find the spectral interpolant for the function whose values on the grid are given by the components of those vectors.

Problem 3

Use Fourier series to find a 2π-periodic solution of wave equation,

$$u_{tt} - u_{xx} = \sin(x - t),$$
$$u_t(x, 0) = e^{ik_0 x}, \; u(x, 0) = 0.$$

Problem 4

Write the Lax-Friedrichs and Lax-Wendroff methods for transport equation. Derive the modified PDEs that are approximated best by each method.

Problem 5

Consider Poisson equation,

$$u_{xx} = f(x),$$
$$u(0) = 0, \; u(1) = 0$$

(a) Consider a discretization of the interval $x \in [0, 1]$ with step $h = 1/4$ and use central-difference formula to approximate the second derivative. Write the associated linear system to determine u at collocation points, and solve it by a direct method.
(b) Write the Jacobi, Gauss-Seidel and SOR methods for solving the resulting system iteratively. State the convergence criterion for each iterative method.
(c) Find eigenvalues for matrices associated with Jacobi and Gauss-Seidel methods. Will these methods converge? Which method converges faster, and why?

Problem 6
Derive the coefficients of spectral differentiation matrix with $N = 4$. Use the matrix obtained to determine the derivative of a function whose values at gridpoints are $(0, 1, 0, -1)$. Find the spectral interpolant and check that its slope at the gridpoints matches your result.

Problem 7
Find the matrix exponential for the matrices in problem 1.