MTH 538 Homework 3

Problem 1
Consider the backward differences in time, and central differences in space stencil for the heat equation,

$$u_t = u_{xx}$$

- Determine the order of the scheme in space and time.
- Apply Von Neumann method to determine the stability properties of the resulting method.

Problem 2
Apply the Von Neumann method to determine stability for upwind and downwind stencil for the transport equation,

$$u_t + cu_x = 0 \quad \text{with} \quad c > 0$$

Determine the local truncation for each stencil, and state the optimal Courant number for the stable scheme.

Problem 3
Consider the Lax-Friedrichs method for transport equation, $u_t = cu_x$,

$$\frac{U_j^{n+1} - \frac{1}{2}\left(U_{j+1}^n - U_{j-1}^n\right)}{\tau} = c\frac{U_{j+1}^n - U_{j-1}^n}{2h}$$

Determine the order of the method in space and time.

Problem 4 Consider IBVP, $u_t = u_x$ with $u(x,0) = \cos x$ and $u(\pi,t) = -\cos t$ on $x \in [0,\pi]$ and $t \in [0,\pi]$.

- Find the exact solution.
- Implement Lax-Friedrichs and Lax-Wendroff methods to solve IBVP.
- Compare the numerical solution at $t = \pi$ to the exact solution for 5 distinct values of the spatial step, h, to show the order of convergence in space for each method.
- Compare the numerical solution at $t = \pi$ to the exact solution for 5 distinct values of the time step, τ, to show the order of convergence in time for each method.
- Explain in your own words how the numerical solutions differ from the exact solution.