MTH 538 Homework 1 Problem 1

Consider the explicit midpoint rule scheme for,

- (1) y' = f(t, y),
- (2) $y(t=0) = y_0,$

and derive explicit formula for the local truncation error, estimate the global truncation error and state the convergence order of the method.

Problem 2

Consider the IVP for the second order equation:

$$u'' + 9u = \sin t,$$

 $u(t = 0) = 1,$
 $u'(t = 0) = 0.$

- Reduce to first order system $\mathbf{y}' = \mathbf{f}(\mathbf{t}, \mathbf{y})$, and obtain exact solution of the IVP, $\mathbf{y}_{exact}(t)$.
- Use the forward Euler method to obtain approximate solutions on the interval $0 \le t \le 2\pi$.
- Use the explicit midpoint rule to obtain approximate solutioons on $0 \le t \le 2\pi$.
- Repeat each solution scheme for timestep sizes $h = \frac{2\pi}{N}$ for N = 50, 100, 200, 400, 800, and evaluate the solution error of each scheme, $\text{Err}(h) = |y_N y_{exact}|$.
- Plot the solution error Err versus h to show order of convergence for each method. (Hint: You should use double logarithmic scale for the plot, where a power law h^p is represented by a straight line).