1. Section 2.5, Computer Problem 5(a), p. 116 of Sauer's textbook.

2. Exercise 2.5.2(c), p. 116 of Sauer's textbook.

3. How many floating point operations does it take to compute

1. the product $A\mathbf{x}$, where A is the sparse $n \times n$ matrix in Problem **1**?

2. the product $B\mathbf{x}$, where B is a full $n \times n$ matrix?

In both cases assume $\mathbf{x}$ is an $n \times 1$ column vector.

4. This problem concerns polynomial interpolation based on expansion

$$p(x) = c_1 + c_2 x + c_3 x^2 + \cdots + c_n x^{n-1}$$

in the monomial basis in tandem with the Vandermonde approach.

(a) Assume that the data $\{(x_k, y_k) : k = 1, \ldots, n\}$ is given, where the nodes x_k are distinct. Derive the linear system $V\mathbf{c} = \mathbf{y}$ that uniquely determines the coefficients c_k.

(b) Write a MATLAB® function (calling sequence `function c = interpvandmon(x,y)`) that returns the vector of expansion coefficients $\mathbf{c} = (c_1, \ldots, c_n)^T$ defining the interpolant, given input vectors $\mathbf{x} = (x_1, \ldots, x_n)^T$ and $\mathbf{y} = (y_1, \ldots, y_n)^T$. Use your function to reproduce the plot given in the notes for the data set $D_4 = \{(0, 1); (1, 4); (2, 1); (3, 1)\}$.

(c) Find the infinity-norm condition number of the matrix V for $n = 5, 10, 15, 20$ equally spaced points on the interval $[0, 1]$.

1. Exercises 3.1.1(c) and 3.1.2(c), p. 149 of Sauer's textbook.

2. Use the MATLAB® function

```
[frame=single,framerule=0.2pt,framesep=5pt]
  function c=interpnewt(x,y)
% function c=interpnewt(x,y)
% computes coefficients c of Newton interpolant through (x_k,y_k), k=1:length(x)
n=length(x);
for k=1:n-1
   y(k+1:n)=(y(k+1:n)-y(k))./(x(k+1:n)-x(k));
end
c=y;
```

along with your own routine for evaluation (calling sequence `p=hornernewt(c,x,z)`, see the rewrite of Sauer's `nest` from the class website) to find and plot the polynomial that is zero at $x = 1, 2, 4, \ldots, 11$ and satisfies $p(3) = 1$. Note that this is the Lagrange basis function $L_3(x)$ for the set of nodal points $\{x_j\}_{j=1}^{11} = \{1, 2, 3, \ldots, 11\}$.

3. Use your routines from **Problem 2** to interpolate the data sets $\{(x_j, y_j) : j = 1, \ldots, n\}$, where $y_j = f(x_j)$ with $f(x) = 1/(x^2 + 1)$.

1. $x_j = -4 + 8\dfrac{j-1}{n-1}$, $n = 11$.
2. $x_j = 4\cos\dfrac{\pi(2j-1)}{2n}$, $n = 11$.

In each case, return 3 plots:

- The data and the interpolant on $[-4, 4]$. The interpolants should be plotted on a dense collection of points, say 500 equispaced points. Include on your plots markers of some kind that illustrate the data points (for example, a MATLAB® plot option like `'r.'`).
- The error $e(x) = f(x) - p(x)$ for $x \in [-4, 4]$.
- The function $g(x) = \dfrac{1}{n!}\displaystyle\prod_{k=1}^{n}(x - x_k)$ for $x \in [-4, 4]$.

Discuss your results.

4. Exercise 3.2.4, p. 156 of Sauer's textbook.