1 HW #3

1. The roots of the quadratic equations $ax^2 + bx + c = 0$ are $x_\pm = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. We may also approximate these roots using fixed-point iterations as follows:
(i) Divide through by x to get the equivalent equation (assuming $x \neq 0$):

$$ax + b + \frac{c}{x} = 0.$$

(ii) Method 1, forward iteration: consider the sequence

$$x_0 = x_{\text{initial}}, \qquad x_{k+1} = -\frac{b}{a} - \frac{c}{ax_k}, \quad k = 1, 2, 3, \ldots.$$

(iii) Method 2, backward iteration: consider the sequence

$$x_0 = x_{\text{initial}}, \qquad x_k = -\frac{b}{a} - \frac{c}{ax_{k+1}}, \quad k = 1, 2, 3, \ldots.$$

that is

$$x_0 = x_{\text{initial}}, \qquad x_{k+1} = -\frac{c}{b + ax_k}, \quad k = 1, 2, 3, \ldots.$$

Write MATLAB® functions `forward.m` and `backward.m` which implement the above iterations to return r which differs from either x_+ or x_- by no more than a tolerance `tol`. Your functions should have `(a,b,c,x0,tol)` as argument list, and terminate when $|x_{k+1} - x_k| \leq \mathsf{tol}/2$. Demonstrate the performance of your functions for the values

$$x_0 = 1, \quad a = 1, \quad b = c = -1, \quad \mathsf{tol} = 10^{-10}.$$

How many iterations are required to reach the desired accuracy? Note that the two functions will return different approximate roots, r_{forward} and r_{backward}. Verify directly that each is within the desired tolerance from either x_+ or x_-. Which root is found by each method? Does the answer change if you change x_0? Now, use r_{forward} as the x_0 for `backward.m` with $\mathsf{tol} = 10^{-10}$. What r do you end up with? Does the answer change if you run `backward.m` again $x_0 = r_{\text{forward}}$, as before, but now with $\mathsf{tol} = 10^{-13}$? Why?

2. Use the bisection method and the MATLAB® function `fzero` to compute a positive real number x satisfying $\sinh x = \cos x$. For each method use a tolerance of 10^{-8}. List your initial approximation (or interval in the case of bisection) and the number of required iterations. Also print at least nine digits of the approximate roots.

3. Use the bisection method and the MATLAB® function `fzero` to compute all three real numbers x satisfying $5x^2 - e^x = 0$. For each of the three roots and each method, use a tolerance of 10^{-8} and list both your initial approximation (or interval in the case of bisection) and the number of iterations needed. Also print at least nine digits for each approximate root.

4. Sauer, page 42, Exercise 24.