1. First, $x = (1011.\overline{010})_2$ has integer part $x_{\text{int}} = (1011.0)_2 = 11$ and fractional part $x_{\text{frac}} = (0.\overline{010})_2$.

$$8x_{\text{frac}} = (010.\overline{010})_2 \implies 7x_{\text{frac}} = (010.0)_2 = 2 \implies \boxed{x_{\text{frac}} = 2/7} \implies \boxed{(1011.\overline{010})_2 = 11\tfrac{2}{7} = \tfrac{79}{7}}$$

Second, $x = (11.101\overline{01})_2$ has integer part $x_{\text{int}} = (11.0)_2 = 3$ and fractional part $x_{\text{frac}} = (0.101\overline{01})_2$.

$$8x_{\text{frac}} = (101.\overline{01})_2 \text{ and } 32x_{\text{frac}} = (10101.\overline{01})_2 \implies 24x_{\text{frac}} = (10101.0)_2 - (101.0)_2 = 21 - 5 = 16.$$

Therefore, $\boxed{x_{\text{frac}} = 16/24 = 2/3}$ and $\boxed{(11.101\overline{01})_2 = 3\tfrac{2}{3} = \tfrac{11}{3}}$.

2. Clearly $2.75 = (10.11)_2 = (1.011)_2 \times 2^1$. Therefore, since 2.75 is associated with a mantissa that is exactly represented in 52 binary digits (indeed, exactly in only 3 binary digits), $\text{fl}(2.75) = 2.75$ exactly, in which case

$$\left|\frac{\text{fl}(2.75) - 2.75}{2.75}\right| = 0 < \tfrac{1}{2}\varepsilon_{\text{mach}}$$

clearly holds. To get the machine representation, consider

$$(1.011)_2 = +1.\underbrace{\boxed{0110\ \ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000}}_{52 \text{ bits}} \times 2^1.$$

The sign bit is 0, because 2.75 is positive. The true exponent is $1 = F - 1023$, so $F = 1024 = (10000000000)_2$, and therefore, the machine representation is

$$\boxed{0 \,\big|\, 100\ 0000\ 0000 \,\big|\, 0110\ \ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000},$$

that is 4006000000000000 in hex format (verify in MATLAB).

3. Let $a = 3344556600$ and $b = 1.2222222$ be the lengths of the right triangle's sides. Its hypotenuse is then

$$c = \sqrt{a^2 + b^2} = \sqrt{3344556600^2 + 1.2222222^2}$$

At the command line in OCTAVE we find

```
octave:10> a=3344556600
a =  3.34455660000000e+09
octave:11> b = 1.2222222
b =  1.22222220000000e+00
octave:12> sqrt(a^2+b^2)-a
ans =  0.00000000000000e+00,
```

that is no digits of accuracy for the naive evaluation. However, notice that

$$c - a = \sqrt{a^2 + b^2} - a = \frac{b^2}{\sqrt{a^2 + b^2} + a} = \frac{b^2}{c + a},$$

or from the OCTAVE command line

```
octave:13> b^2/(sqrt(a^2+b^2)+a)
ans =  2.23322144731059e-10
```

This answer is accurate to essentially all reported digits.

4. For **(a)** the MATLAB function is as follows.

```
%  Function obtains roots of
%        a x^2 + b x + c = 0
%  by direct use of quadratic formula.
%  function [xm xp] = SimpleQuadratic(a,b,c)
   function [xm xp] = SimpleQuadratic(a,b,c)
xm = 0.5*(-b - sqrt(b^2 - 4*a*c))/a;
xp = 0.5*(-b + sqrt(b^2 - 4*a*c))/a;
```

For (i) the roots are $x_{\mp} = -\frac{3}{4} \mp \frac{1}{4}\sqrt{9-8} = -1, -\frac{1}{2}$. Using MATLAB, we find the following.

```
>> a = 2; b = 3; c = 1;
>> [xm xp] = SimpleQuadratic(a,b,c);
>> disp([xm xp])
    -1.000000000000000e+00    -5.000000000000000e-01
```

For (ii) the roots are $x_{\mp} = -\frac{3}{2} \mp \frac{1}{2}\sqrt{9-16} = -\frac{3}{2} \mp \mathrm{i}\frac{\sqrt{7}}{2}$. Using MATLAB, we find the following (some spaces removed from command window output).

```
>> a = 1; b = 3; c = 4;
>> [xm xp] = SimpleQuadratic(a,b,c);
>> disp([xm xp])
  Column 1
     -1.500000000000000e+00 - 1.322875655532295e+00i
  Column 2
     -1.500000000000000e+00 + 1.322875655532295e+00i
```

Note `sqrt(7)/2 = 1.322875655532295e+00`. So the simple quadratic formula yields accurate roots for (i) and (ii).

For **(b)** and case (iii) the exact roots are

$$x_{\mp} = -\tfrac{3}{2} \mp \tfrac{1}{2}\sqrt{9 - 4 \times 8^{-14}}.$$

We will use MATLAB's `root` function to define the "exact" roots and compare our answers with those obtained with this internal function. We use both the function `SimpleQuadratic.m` given above, as well as the following one.

```
%  Function obtains roots to quadratic equation
%        a x^2 + b x + c = 0
%  by quadratic formula, but with modification to improve
%  accuracy. Here the modification is relevant for b > 0
%  with sqrt(b^2 - 4*a*c) close in size to b. Namely,
%
%  xm = -(b + sqrt(b^2 -4ac))/(2a) ... usual formula.
%  xp =  c/(a*xm) ..................... modified formula.
%
%  Note modified formula for xp avoids subtraction of
%  similar size numbers which is prone to accuracy loss.
%  function [xm xp] = ModifiedQuadratic(a,b,c)
   function [xm xp] = ModifiedQuadratic(a,b,c)
xm = -0.5*(b + sqrt(b^2 - 4*a*c))/a;
xp =  c/(a*xm);
```

To generate and compare the outputs, we have used the following script.

```
% Script: Set2Problem4b
% Makes table required by Problem 4b of Homework Set 2.

a = 1; b = 3; c = 8^(-14);
```

```
% Matlab computation of the roots.
tmp = roots([a b c]);
xm_matlab = tmp(1); xp_matlab = tmp(2);

% Roots obtained via naive use of quadratic formula.
[xm_simple xp_simple]  = SimpleQuadratic(a,b,c);

% Roots obtained with modification of quadratic formula.
[xm_modified xp_modified]  = ModifiedQuadratic(a,b,c);

FID = fopen('Set2Problem4b-Table.txt','w');
fprintf(FID,'       SimpleQuadratic         ModifiedQuadratic       roots([1 3 8^(-14)])\n');
fprintf(FID,'-------------------------------------------------------------------------------\n');
fprintf(FID,'x_m: %1.15e  %1.15e  %1.15e\n',xm_simple,xm_modified,xm_matlab);
fprintf(FID,'x_p: %1.15e  %1.15e  %1.15e\n',xp_simple,xp_modified,xp_matlab);
fclose(FID);
```

The script outputs the table

```
       SimpleQuadratic         ModifiedQuadratic       roots([1 3 8^(-14)])
-------------------------------------------------------------------------------
x_m: -2.999999999999925e+00  -2.999999999999925e+00  -2.999999999999924e+00
x_p: -7.571721027943568e-14  -7.579122514774592e-14  -7.579122514774593e-14
```

to the file `Set2Problem4b-Table.txt`. Our modified algorithm yields a second root in excellent agreement with MATLAB's answer, while the naive formula for x_+ does not.

If student made a mistake, using instead of 8^{14} value $8e-14$, please, do not subtract points. In this case right roots are:

```
>> roots([1 3 8e-14])

ans =

    -2.999999999999973e+00
    -2.666666666666690e-14
```