1. Answers obtained by inspection[1], and only afterward checked in MATLAB.

```
q=[0 40 0 80 0 90 0 5]
q=[0 40 0 80 0 90 0 5]
q=[0 40 0 0 30 90 10 0]
```

2. Again, answers obtained by inspection, and only afterward checked in MATLAB.
(a) (i) `t=linspace(5,30,6)`, (ii) `x=linspace(-3,3,7)`
(b) (i) `v=[-2:0.75:1]`, (ii) `r=[6:-1:0]`

3. Assume that the N in the summations is `length(x)`. Not specified in the problem! (i) `sum(x)`, (ii) `x*transpose(y)` or `x*y'` (since `y` is real), (iii) `x*transpose(x)` or `x*x'` or `sum(x.^2)`.

4. The input matrix should have been (note trailing semicolons to start new lines).

```
a = [ 0.035    0.0001    10      2;
      0.020    0.0002     8      1;
      0.015    0.0010    20      1.5;
      0.030    0.0007    24      3;
      0.022    0.0003    15      2.5];
n=a(:,1); S=a(:,2); B=a(:,3); H=a(:,4);
```

With this input, at the command line we have the following (output edited to remove empty lines).

```
>> format short g
>> u=sqrt(S)./n.*(B.*H./(B+2*H)).^(2/3)
u =
      0.36241
      0.60937
       2.5167
       1.5809
       1.1971
```

Note `format short g` is appropriate here, since the input data is only known to 2 significant figures. Might also report fewer digits.

5. For part **(a)** we have used the following script.

```
% Script: Set1Problem5a
% Makes plots required by Problem 5a of Homework Set 1.

x = linspace(-1,1,100);
h = 1e-4;
fp_exact = -2*x./(1+x.^2).^2; % Exact derivative of f(x) = 1/(1+x*x).

% First subplot
fp = onesidediff(@bellshape,x,h);
subplot(2,1,1)
explot(x,abs(fp-fp_exact))
xlabel('x')
ylabel('| (df/dx)_e_x_a_c_t - (df/dx)_a_p_p_r_o_x |')
title('Error between exact derivative of 1/(1+x^2) and one-sided stencil')

% Second subplot
fp = centerdiff(@bellshape,x,h);
subplot(2,1,2)
explot(x,abs(fp-fp_exact))
xlabel('x')
ylabel('| (df/dx)_e_x_a_c_t - (df/dx)_a_p_p_r_o_x |')
```

[1] Could you do the same, say on a test?

```
title('Error between exact derivative of 1/(1+x^2) and centered stencil')

% Save figure as an eps.
saveas(gcf,'Set1Problem5a.eps','epsc')
```

The plots made by the script are shown in Fig. 1. See the figure caption for more information.

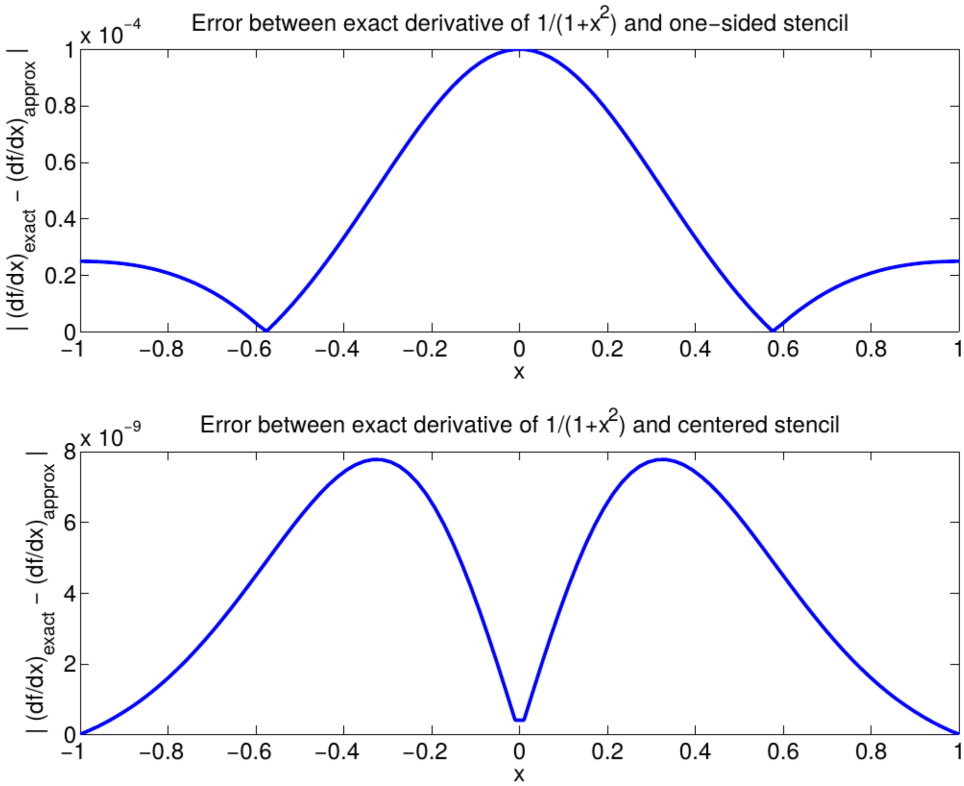

Figure 1: Plots for Problem 5a. Here we plot the error in the two approximations over 100 uniformly spaces points on $[-1, 1]$. Each approximation uses the spacing $h = 10^{-4}$.

For parts **(b)** and **(c)** we have used the following script.

```
% Script: Set1Problem5bc
% Makes table and plot required by Problem 5b,c of Homework Set 1.

h = transpose(logspace(-1,-9,9));
DataValues = zeros(length(h),5);
DataValues(:,1) = h;
for k=1:length(h)
    fp = onesidediff(@exp,0,h(k));
    DataValues(k,2) = fp;
    DataValues(k,3) = abs(fp-1);
    fp = centerdiff(@exp,0,h(k));
    DataValues(k,4) = fp;
    DataValues(k,5) = abs(fp-1);
```

```
end

% Make the table for part b.
FID = fopen('Set1Problem5b-Table.txt','w');
fprintf(FID,'------------------------------------------------------------------------\n');
fprintf(FID,'|    h    | one-sided approx |   error    | centered approx  |   error    |\n');
fprintf(FID,'------------------------------------------------------------------------\n');
for k = 1:length(h)
  fprintf(FID,'| %1.1e | %1.14f | %1.3e | %1.14f | %1.3e |\n', ...
          DataValues(k,1),DataValues(k,2),DataValues(k,3),DataValues(k,4),DataValues(k,5));
end
fprintf(FID,'------------------------------------------------------------------------\n');
fprintf(FID,'   Errors taken in absolute value with respect to (df/dx)(0) = e^0 = 1   \n');
fclose(FID)

% Make the plot for part c.
figure(2)
hold off
exloglog(DataValues(:,1),DataValues(:,3),'k--o')
hold on
exloglog(DataValues(:,1),DataValues(:,5),'b-d')
legend('one-sided approx','centered approx','Location','NorthWest')
title('Errors in approximation of (df/dx)(0) for f(x) = e^x')
xlabel('h')

% Save figure as an eps.
saveas(gcf,'Set1Problem5c.eps','epsc')
```

The table required by part **(b)** is as follows (note that the above script is configured to save this table as a text file called `Set1Problem5b-Table.txt`).

h	one-sided approx	error	centered approx	error
1.0e-01	1.05170918075648	5.171e-02	1.00166750019844	1.668e-03
1.0e-02	1.00501670841679	5.017e-03	1.00001666674999	1.667e-05
1.0e-03	1.00050016670838	5.002e-04	1.00000016666668	1.667e-07
1.0e-04	1.00005000166714	5.000e-05	1.00000000166689	1.667e-09
1.0e-05	1.00000500000696	5.000e-06	1.00000000001210	1.210e-11
1.0e-06	1.00000049996218	5.000e-07	0.99999999997324	2.676e-11
1.0e-07	1.00000004943368	4.943e-08	0.99999999947364	5.264e-10
1.0e-08	0.99999999392253	6.077e-09	0.99999999392253	6.077e-09
1.0e-09	1.00000008274037	8.274e-08	1.00000002722922	2.723e-08

Errors taken in absolute value with respect to (df/dx)(0) = e^0 = 1

Figure 2 depicts the corresponding errors in a loglog plot. We observe that for both approximations the error drops initially (this drop is proportional to h for the one-sided difference and to h^2 for the centered difference). However, in both cases convergence to the true value $f'(0) = e^0 = 1$ stalls for small h (due to the finite-precision of computer arithmetic, as we will learn later).

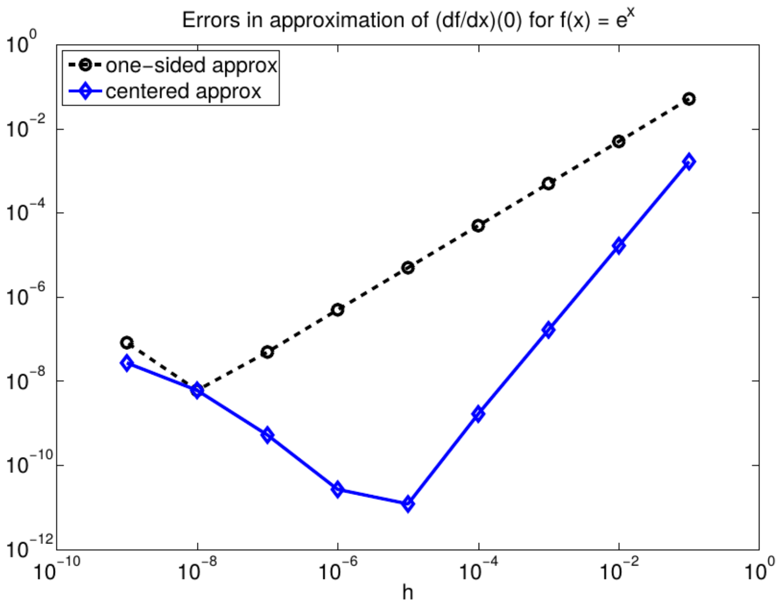

Figure 2: Plot for Problem 5c.