

— HOMEWORK #1

Remember to clearly label all plots and include any MATLAB scripts and functions with your solution. Also please remember that no late homeworks will be accepted.

1. Suppose $q=[50 \ 40 \ 60 \ 80 \ 30 \ 90 \ 10 \ 5]$. What does this vector look like after each of these commands?

$q(1:2:7)=\text{zeros}(1,4)$

$q(7:-2:1)=\text{zeros}(1,4)$

$q([3 \ 4 \ 8 \ 1])=\text{zeros}(1,4)$

2. (a) Use the `linspace` function to create vectors identical to the following created with colon notation:

(i) $t=5:5:30$

(ii) $x=-3:3$

- (b) Use colon notation to create vectors identical to the following created with the `linspace` function:

(i) $v=\text{linspace}(-2,1,5)$

(ii) $r=\text{linspace}(6,0,7)$

3. Given that $x=0:.01:1$; $y=\cos(x)$; , write a single-line MATLAB code that returns the following.

(i) $\sum_{k=1}^N x_k$ (use the `sum` routine)

(ii) $\sum_{k=1}^N x_k y_k$ (do not use the `sum` routine)

(iii) $\sum_{k=1}^N x_k^2$

4. Manning's equation can be used to compute the velocity of water in a rectangular open channel:

$$U = \frac{\sqrt{S}}{n} \left(\frac{BH}{B+2H} \right)^{2/3},$$

where U = velocity (meters/sec), S = channel slope, n = roughness coefficient, B = width (meters), and H = depth (meters). The following data is available for five channels.

n	S	B	H
0.035	0.0001	10	2
0.020	0.0002	8	1
0.015	0.0010	20	1.5
0.030	0.0007	24	3
0.022	0.0003	15	2.5

Given the MATLAB code

```
a=[ 0.035    0.0001    10     2
    0.020    0.0002     8     1
    0.015    0.0010    20    1.5
    0.030    0.0007    24     3
    0.022    0.0003    15    2.5];
n=a(:,1); S=a(:,2); B=a(:,3); H=a(:,4);
```

Write a single-line MATLAB statement to compute a column vector containing the velocity based on the values in the parameter matrix.

5. Consider the following *finite difference quotients*

$$\text{(one-sided formula)} \quad \frac{f(x+h) - f(x)}{h}, \quad \text{(centered formula)} \quad \frac{f(x+h) - f(x-h)}{2h},$$

each of which approximates the derivative $f'(x)$ of a differentiable function f .

(a) Write two MATLAB functions, with calling sequences (fp for f prime) `fp = onesidediff(f,x,h)` and `fp = centerdiff(f,x,h)`, which evaluate these quotients on a vector x , given a scalar h and a general function f passed using the `@` notation. Use both MATLAB functions to approximately compute the derivative of $f(x) = (1+x^2)^{-1}$ on $[-1, 1]$ using `x = linspace(-1,1,100)` and `h = 1e-4`. Prepare the following plot with two subplots, using the `subplot(2,1,1)` and `subplot(2,1,2)` commands. The top plot should depict the error (in absolute value) on $[-1, 1]$ between the one-sided approximation and the exact derivative, while the bottom plot should depict the error (in absolute value) on $[-1, 1]$ between the centered approximation and the exact derivative.

(b) For $f(x) = e^x$ and h in the range $[10^{-1}, 10^{-2}, \dots, 10^{-9}]$, compute $f'(0)$ (the exact value is obviously $e^0 = 1$) using the MATLAB function from (a). Make a table which lists h in the first column, the one-sided approximation in the second column, the error in the one-sided approximation in the third column, the centered approximation in the fourth column, and the error in the centered approximation in the fifth column. Report h in scientific notation with a minimum number of displayed digits. Report errors in absolute value using scientific notation keeping 3 digits past the decimal. Approximations should be reported in fixed-point non-scientific notation with a full field of digits (say 14 past the decimal point).

(c) Plot the errors from (b) versus h on the same log-log plot. Your plot should include a legend. What do you observe?