

Extra Qual Exam Review

Problem 1

Consider the values of a 2π -periodic function on a uniform grid with $N = 4$ grid points,

$$x_j = jh, \quad j = 0, 1, 2, 3, \quad h = \frac{2\pi}{N}.$$

Suppose the sampled values are

$$u(x_0) = 1, \quad u(x_1) = 3, \quad u(x_2) = 1, \quad u(x_3) = -1.$$

(a) Compute the discrete Fourier coefficients \hat{u}_k for

$$k = -1, 0, 1, 2.$$

(b) Using these Fourier coefficients, construct the interpolating trigonometric polynomial

$$p(x) = \sum_{k=-1}^2 \hat{u}_k e^{ikx}.$$

(c) Draw the graph of $p(x)$ on $[0, 2\pi]$ and mark the four interpolation points.

(d) Verify directly that

$$p(x_j) = u(x_j), \quad j = 0, 1, 2, 3.$$

Note:

I decided that there will not be a problem on numerical method for eigenvalue problem solution because we already have plenty of material as is.