A Mathematical Model for the Prediction of Speeding with its Validation

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Abstract—Speeding is one of the most prevalent contributing factors in traffic crashes. The prediction of speeding is important for reducing excessive speeds and preventing speeding-related traffic accidents and injuries. Speeding (either intentional or unintentional) is a consequence of inappropriate speed control. This work extends a previous mathematical model of driver speed control to provide quantitative predictions of intentional and unintentional speeding. These predictions consist of the time at which the driver exceeds the speed limit and the magnitude of speeding. Based on these modeling predictions, this work develops an intelligent speeding prediction system (ISPS) to prevent the occurrence of speeding. An experimental study using a driving simulator is conducted to evaluate ISPS. We find no significant difference between modeled predictions and experimental results in terms of the time and magnitude of intentional speeding. Also, ISPS can successfully predict the majority of unintentional speeding instances, with only a small portion of unnecessary speeding warnings (false alarms). Applications of the ISPS to reducing driving speed, and preventing the real-time occurrence of speeding and speeding-related traffic accidents are discussed.

Index Terms—Speeding, speeding prediction, mathematical model, in-vehicle intelligent system.

I. INTRODUCTION

SPEEDING (exceeding the posted speed limit, racing or driving too fast for conditions) is one of the most prevalent contributing factors in traffic crashes in the United States and many other countries. In 2009, speeding contributed to 31% of all fatal crashes in the United States, which resulted in the loss of 10,591 lives [1]. More importantly, speeding is common and even universal on some roads. The National Highway Traffic Safety Administration (NHTSA) conducted a survey in 2002 and reported that 80 percent of all drivers exceeded the posted speed limit during the month before the survey was taken. These drivers believed that they could drive about 7-8 mph over the posted speed limit before they will be ticketed [2].

Many strategies consisting of infrastructural (e.g., speed bumps, roundabouts) or legislative interventions (e.g., reduced speed limit, higher fines for speeding violation) are adopted to improve speed limit compliance and reduce excessive speeds. In-vehicle speed assistance systems are one of these strategies. Intelligent speed adaptation (ISA) is a typical speed assistance system that compares the vehicle’s current speed with the posted speed limit and provides the driver with visual, auditory and/or tactile warnings, or limits the vehicle’s speed if the driver exceeds the speed limit. Much research has examined the safety benefits of ISA technologies and their influence on driving performance in different countries, generally finding positive effects on average speed [3-6].

Similar video-based speed surveillance system that monitors and detects speeding via traffic cameras rather than sensors has been developed [7-9]. In essence, such speed surveillance system and ISA are post-feedback systems that provide the driver with a warning message after he/she exceeds the speed limit beyond a specified threshold (e.g., by 1 mph). These post-feedback systems may be too late to warn a driver when he/she is already speeding, particularly in some emergent situations. For example, a driver is speeding and, at this moment, a lead vehicle suddenly stops or a pedestrian suddenly crosses the street. Although ISA warns the driver about his/her travelling speed, it may be too late for the driver to stop or avoid a collision. Since the braking distance and kinetic energy are positively proportional to the square of the driving speed, the possibility of a collision, as well as collision severity, always becomes larger as the speed increases. This phenomenon was also confirmed by previous studies with real traffic data [10-12]. Elvik et al. [12] estimated that a 1% increase in speed resulted in a 3.7% increase in fatal accidents and 2% increase in accidents with injuries. Therefore, a driver may benefit more from a new speed assistance system that can predict the occurrence of speeding violations better than the previous ISA systems.

Speeding can be classified into two categories: intentional and unintentional [5]. Intentional speeding refers to the intention or motivation to speed. For example, a driver speeds because of time-pressure or enjoyment of driving fast. A few theoretical approaches have been developed to predict an intentional speeding violation. The theory of planned behavior (TPB) is the most widely used theoretical framework. TPB describes a person’s attitude towards his/her own behavior, subjective norm, perceived behavioral control, and moral norm as the four major components that determine a person’s behavior based indirectly upon their intentions. The more positive a person’s attitude, subjective norm, and moral norm, and the greater their perceived control are, the stronger their intention is to exhibit the specified behavior. TPB has been used to predict speeding in the past, and it was reported that intention was the prominent predictor of observed speeding behavior: TPB model can explain about 45-55% variance in intention to speed [13, 14]. However, because TPB only captures the conscious aspect of speeding, it cannot account for or predict the occurrence of an unintentional speeding.

Unintentional speeding may result from a lack of awareness of the current speed limit and/or travelling speed [5]. Also, foot movement required for pedal operation is not always perfectly...
executed, and thus produces errors. Compared to hands, feet are slower and less accurate in control operation [15, 16]. For example, some researchers observed that drivers’ feet tend to deviate from their original postures and produce a small error over time [17, 18]. Fuller [19] developed a theoretical approach, the task capability interface model (TCI), to study driver behaviors. TCI continuously compares capability (C) with task demand (TD) at each sampling time point. Based on the TCI model, Brandenburg and Drewitz [20] propose a mathematical equation to describe the amount of vehicle acceleration as a function of the difference between C and TD. But, these authors do not validate the model’s predictions of speeding (such as the time at which a driver exceeds the speed limit or the magnitude of speeding) with experimental or empirical data in their study.

Speeding is a consequence of inappropriate speed control. Speed control involves a series of time-sharing activities, including speed perception, decision making, and speed adjustment. Zhao and Wu [21] developed a mathematical model of driver speed control. This model integrates four important aspects of speed control in a cohesive manner: speed perception, decision making, motor control, and vehicle mechanics. A driver continuously perceives and estimates travelling speed while his/her vehicle is in motion. Posted speed limits serve as stimuli that trigger a driver’s cognitive process of making a decision regarding his/her desired target speed, regardless of the effects of other vehicles (i.e., free-flow driving). Upon realizing the difference between their target speed and perceived speed, drivers react by moving their feet accordingly in order to operate the pedal. Foot movement causes the pedal input’s changes, which eventually lead to the changes of vehicle acceleration and speed. Because this model provides quantitative predictions of vehicle acceleration and speed, it is able to predict speeding. However, because this model requires the cognitive behavior of decision making (i.e., setting a target speed based on the current posted speed limit), it only predicts the occurrence of intentional speeding. Also, when a driver accelerates/decelerates to his/her target speed and eventually maintains a constant speed, speeding may occur due to pedal control errors. But the previous model does not account for this phenomenon.

In addition, the previous model of driver speed control is applied to the average driver but not to individual drivers. Based on the literature, two major individual factors have been examined in existing studies on driver speed control: decision making references (DMR) and impulsiveness. Each driver has his/her own decision making reference that is derived and modified during previous driving instances. Some drivers believe that they can drive about 7-8 MPH over the posted speed without being caught [2], while others may prefer to follow the speed limit. Such decision making references differ according to changes in speed limit, road type, weather condition, traffic flow, and social environmental variables, such as surrounding drivers [22]. Impulsiveness is another salient factor that has received attention in the accident prevention literature but not in the previous speed control models. Impulsiveness is one component of risk-taking, and it is associated with driving anger [23] and most aggressive driving, such as speeding [21]. It deals with a person’s control over his/her thought and behavior, and it may lead to speeding behavior if an individual lacks the self-control to refrain from engaging in such an action [24]. Because the prediction of speeding for individual drivers is more crucial than that of drivers in general, it is necessary to add a new component in the current model to capture individual differences.

The objectives of this study are to 1) Extend the previous model of driver speed control to provide quantitative predictions of both intentional and unintentional speeding for individual drivers, and 2) Design an intelligent speeding prediction system to prevent the occurrence of speeding based on this new extended work.

II. THE NEW INTELLIGENT SPEEDING PREDICTION SYSTEM

In this paper, a new intelligent speeding prediction system (ISPS) is developed to predict the time at which a person drives over the speed limit and the magnitude of speeding. As illustrated in Fig. 1, the basic architecture of the ISPS consists of three major elements: 1) Model inputs and associated in-vehicle technology; 2) A mathematical model and data processing module; 3) Warning messages with Human Machine Interface.

A. Model Inputs and Associated Technology

Mechanical sensors installed in the vehicle measure the dynamic vehicle variables including speed, acceleration, throttle, and brake pedal inputs. These variables are readily available from existing in-vehicle sensors, and using them avoids unnecessary costs related to installing additional sensors. In addition to vehicle variables, in-vehicle GPS, video, and
other technologies are required to measure dynamic environmental variables. Such variables consist of the formation of the current speed limit (the location and the level of the speed limit), the density of outside texture, traffic flow, and signal light. Finally, driver characteristic is another source of model inputs consisting of individual decision making reference and personal impulsiveness. These three categories of model input are filtered and transmitted into an in-vehicle computer.

B. Mathematical Model of Driver Speed Control

The model of driver speed control consists of four major components: speed perception, decision making, motor control, and vehicle mechanics

1) Speed Perception: In a real driving situation, most of the time, drivers are aware of their traveling speed relying on perceptual cues, combined with occasional speedometer inspection. These perceptual cues may be visual, auditory, or kinesthetic cues. While each category plays an important role in assessing traveling speed, visual cues (e.g., optical flow), serve as the predominant clues that drivers use to estimate their travelling speed [25, 26]. Compared to speedometer inspection, such subjective estimation is not accurate, causing drivers to tend to underestimate their travelling speed, produce faster speeds to compensate, and therefore lead to speeding behavior [25-27].

Much existing literature on speed perception suggests that a driver’s perception of speed varies directly with a vehicle’s travelling speed, vehicle height, and the density of visible texture [28-31]. Based on the literature, Zhao and Wu [21] propose that the ratio of perceived speed $v_p$ to actual speed $V$ directly vary with the ratio of the current texture density $D_c$ to the texture density in the last driving scenario $D_s$, and the ratio of the eye height in the last driving scenario $H_s$ to the current eye height $H_c$, with two constant parameters $k_1$ and $k_2$ [see Equation 1; 21]. When a driver checks the speedometer, his/her perceived speed is equal to the actual vehicle speed ($v_p=V$).

$$v_p = \left(\frac{D_c}{D_s}\right)^{k_1} \times \left(\frac{H_s}{H_c}\right)^{k_2} \times V$$

2) Decision Making: The previous model of driver speed control incorporates and extends the Rule-based Decision Field Theory (RDFT; 32) to model a driver’s cognitive behavior of speed choice. When new drivers drive in their first few occasions, they have to deliberate for each speed choice using only the attribute information (deliberation process). The attribute information may consist of potential money lost (e.g., get a speeding ticket), time benefit (e.g., save time if driving faster) or driving safety gain. With successive driving, drivers may encounter a set of rules (e.g., slow down when it is snowy) that become applicable when successfully setting a target speed. At this time, simple rule(s) competed with the attributes listed above depending on each driver’s speed preference (learning process). In addition, the preference for each option modifies and accumulates over time. For example, a driver exceeds the speed limit 10 mph and never receives a speeding ticket. After that, s/he may drive 5 mph over the speed limit and stick to this new choice.

RDFT has successfully modeled the cognitive behavior of speed choice by mathematical means. Specifically, each driver has to formulate a subjective value (or attribute) metric $M$ for the possible outcome to demonstrate the cognitive process of speed choice (e.g., follow the speed limit or drive 5 mph over the speed limit). Based on this value metric $M$ along with driver’s attention weight to each attribute/rule $W(t)$, and the advantage of one speed choice relative to all others $V(t)$, RDFT computes the preference values $P(t)$ for all speed choices at each time $t$. These preference values accumulate over time. Once a preference for its associated speed choice (i.e., drive over the speed limit 5 mph) exceeds a certain threshold $\theta$ ($P(t) \geq \theta$), this speed choice is chosen. As a result, the desired target speed $v_{tar}^*(t)$ is equal to the posted speed limit $V_p$ plus the selected speed choice $v_{P(t)}$ [see Equation 2; 21]. For a detailed description of the model development and parameter estimation regarding RDFT, see [21, 32].

$$v_{tar}^*(t) = V_p + v_{P(t)\geq\theta}$$

3) Motor Control: Foot rotates around the heel joint (a so-called pivot) when stepping on the pedal. In a normal (non-emergent) driving situation, drivers are assumed to make comfort pedal operation and do not move their feet beyond the comfort range for their ankle [33, 34]. The previous model of driver speed control propose that the angular speed of the foot movement required for pedal operation $\omega = d\theta / dt$ directly varies with the difference between a driver’s desired target speed $v_{tar}$ and perceived speed $v_p$ at the time $t$, and $A$ is a constant [see Equation 3; 21].

$$\omega = A \times (v_{tar} - v_p)$$

According to Equation 3, it suggests that the difference between a driver’s desired target speed and perceived speed triggers the response of foot movement, and determines the level of foot angular speed.

4) Vehicle Mechanics: Foot movement leads to the changes of pedal inputs, causing the changes of a vehicle’s acceleration and speed. Previous vehicle mechanical models as well as experimental studies suggest that the vehicle speed is a nonlinear function of the pedal input [35]. To simplify this mechanical process, Zhao and Wu [21] propose a simple linear equation to approximately quantify the relation between the deviation of pedal input and the deviation of acceleration. As illustrated in Equation 4, the deviation of the acceleration $da$ directly changes with the deviation of the pedal angle $d\theta$ at each small time interval $\Delta t$, and $B$ is a constant.

$$da = B \times d\theta$$

Then, the vehicle’s acceleration $a$ and speed $V$ applied at the
next time interval is described as follows:

\[ a = a_0 - a_x + da \]
\[ = a_0 - c_x \times V_0^2 + \int_0^\Delta t A \times B \times f \times (v_x^2 - v_p^2) \times dt \]  
(5)

\[ V = V_0 + a\Delta t \]
\[ = V_0 + [a_0 - c_x \times V_0^2 + \int_0^\Delta t A \times B \times f \times (v_x^2 - v_p^2) \times dt] \times \Delta t \]  
(6)

Where, \( a_0 \) and \( c_x \) represent the initial acceleration and the coefficient of the overall drag on the vehicle.

Note that the existing model of driver speed control is able to predict a driver’s speed behavior, such as the time at which a driver exceeds the speed limit and the magnitude of speeding. However, this model requires a cognitive behavior of decision making on speed choice, so it can only predict intentional speeding behaviors. In the following section, we extend the current model to predict both intentional and unintentional speeding considering individual differences in decision making references and impulsiveness.

C. Extension for Driver Speed Control

This section introduces a new model component, individual differences, and extends the model of driver speed control to provide predictions of unintentional speeding.

1) Individual Differences: Two individual differences, decision making references and impulsiveness, are modeled in this work. Firstly, a subjective attribute matrix \( M \) is constructed for each driver. Each row represents an option or speed choice and each column depicts a possible outcome. The numbers at the cross section of each row and column are subjective values that each individual driver associates with the cost/benefit of speeding. Because these numbers are the aggregation of multiple attributes (monetary cost of receiving a speeding ticket, a gain in terms of safety, and saving travel time), no qualifying units are assigned to these values (i.e., no unit of measurement).

Every driver establishes their own individual reference points including the number of row and column, scales, and the values in each cell. For example, as shown in Table I, there are four speed choices: follow the speed limit (no speeding), drive over the speed limit at 5 mph (drive at +5), drive over the speed limit at 10 mph (drive at +10), and drive over the speed limit at or greater than 15 mph (drive at \( \geq +15 \)). Also, there are four possible outcomes: the driver gets a speeding ticket when driving 5 mph over the speed limit (Ticket at +5), 10 mph over (Ticket at +10), and 15 mph over (Ticket at \( \geq +15 \)), and the driver receives no speeding ticket no matter how fast he/she drives (No ticket at all). If the driver drives over the speed limit by 5 mph and gets caught (the case at the cross section of the second row and the first column in Table I), the driver feels that he/she would experience a net loss of -300 from receiving a ticket even he/she gains some time and safety benefits. Based on this attribute matrix, each driver’s desired target speed is calculated.

Secondly, previous empirical studies did not show evidence on the influence of impulsiveness on speed perception; therefore, in this work, we only focus on modeling the effect of impulsiveness on decision making and motor control. Compared to normal drivers, those who are characterized as impulsive or non-impulsive drivers may have a strong preference for attending to a specific attribute (i.e., time benefit). For example, while an impulsive driver is deliberating about how fast to drive, his/her attention may shift to thinking more about the time benefit (driving fast saves time), rather than stochastically drift from one attribute to another in which the probability of attention to an attribute at each moment is equal to the probability (of occurrence) of each outcome [32]. Accordingly, we form a new attention weights matrix \( \Psi(t) \) to represent how impulsive or non-impulsive drivers shift their attention for each outcome. Specifically, the Extroversion (E) and Neuroticism (N) scales of Eysenck’s personality system [36, 37] are used to divide all drivers into three categories: normal drivers (those characterized as E+ and N- or E- and N+), impulsive drivers (those characterized as E+ and N+), and non-impulsive drivers (those characterized as E- and N-). Then, normalized scores1 (ranging from 0 to 1) in E and N scales of Eysenck’s personality system are averaged (represented by \( \Psi \)). For impulsive drivers, \( \Psi \) represents the probability of attention to the time benefit, and the remainder from one is evenly portioned among the other three attribute weights. By contrast, \( (1 - \Psi) \) represents the probability of attention to the speeding cost and driving safety, and \( \Psi \) is evenly distributed among the other three attribute weights for non-impulsive drivers.

Moreover, personality effects on motor speed control have been well established in a variety of tasks [38-40]. Of the variety of personality inventories available, the relations between motor speed and personality are mostly consistent for E and N scales of Eysenck’s personality system with a variety of behavioral paradigms [41]. Bachorowski and Newman [38] reported that impulsive persons (characterized as E+ and N+) had faster motor speed in a task calling for slow and controlled movement compared to non-impulsive ones (characterized as E- and N-), and this effect was more salient when a behavior

<table>
<thead>
<tr>
<th>Table I</th>
<th>Subjective Values in Terms of Monetary Cost, a Gain of Time, and Safety (Further Developed From, [32])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ticket at +5mph</td>
</tr>
<tr>
<td>No speeding</td>
<td>100</td>
</tr>
<tr>
<td>Drive at +5mph</td>
<td>-300</td>
</tr>
<tr>
<td>Drive at +10mph</td>
<td>-250</td>
</tr>
<tr>
<td>Drive at ( \geq +15 )mph</td>
<td>-500</td>
</tr>
</tbody>
</table>

1 In a short form of the Revised Eysenck Personality Questionnaire (EPQR-S, Eysenck et al. 1985), Extraversion and Neuroticism each contain 12 items (ranging from -12 to 12). Normalized scores \( x = x / 24 + 0.5 \) (\( x \): original scores in E and N scales; \( x \): normalized scores in E and N scales).
goal was present. As illustrated by Equation 7, a personality characteristic variable \( \eta \), which represents the degree of impulsivity or the tendency to act rapidly without deliberation \([42]\), is introduced in the Equation 3. Based on Bachorowski and Newman’s findings in their previous study, \( \eta \) is estimated accordingly (0.736 for drivers characterized as E- and N-; 1.533 for E+ and N+; and 1 for E+ and N- or E- and N+).

\[
\omega = \frac{d\theta}{dt} = A \times \eta \times (V_v - v_p) \tag{7}
\]

2) Prediction of Unintentional Speeding: Unintentional speeding behavior does not involve cognitive decision making on speed choice. Therefore, the differences between a driver’s desired target speed and perceived speed cannot trigger the response of foot movement required for pedal operation. Instead, we measure and use the deviation of the throttle pedal input to predict the occurrence of unintentional speeding. The previous study has successfully predicted driving behavior (i.e., the probability of the future stop at a signal intersection) based on pedal strokes \([43]\).

Let \( \theta_{max} \) and \( \theta_{min} \) be the steady state of the maximal and minimal throttle pedal angle in a small time window (i.e., 1s), and \( V_{min} \) and \( t_{max} \) be the corresponding speed and elapsed time with the minimal throttle pedal angle. Define \( d\theta = \theta_{max} - \theta_{min} \) as the difference between the maximal and minimal throttle pedal angle. If such difference exceeds the predefined threshold \( \theta_p \) (e.g., \( \theta_p = 3^\circ \)), the model predicts the time \( t \) at which a driver exceeds the posted speed limit plus a small tolerance \( \Delta V \) (see Equation 8).

\[
t = t_{max} + \frac{V_{at} + \Delta V - V_{min}}{a + B \times d\theta - c_2 \times V_{min}^2} \tag{8}
\]

It is important to note that this new extended model only provides predictions of speeding when the initial speed is under the speeding criteria (see Equation 9).

\[
V_{min} < V_{at} + \Delta V \tag{9}
\]

Based on this model, the data processing module in ISPS predicts the time at which a driver exceeds the speed limit beyond a specified threshold. These time points are recorded and managed for scheduling a speeding warning message. The warning message is played only if the difference between predicted time \( t \) and elapsed time \( t_{at} \) is greater than a specified threshold (see Equation 10). This threshold is determined based on the duration of the warning message \( t_{at} \) and driver reaction time RT. In other words, we have to make sure that a driver has enough time to listen to such a warning message and respond. If a driver hears the warning message and responds when he/she has already exceeded the speed limit, ISPS becomes a post-feedback warning system, such as warning ISA.

\[
\Delta t = t - t_{at} \geq t_{at} + RT \tag{10}
\]

Then schedule a speeding warning message

If the decision making component predicts that a driver’s target speed is equal or below the speed limit, the model only predicts the occurrence of an unintentional speeding for this driver. In contrast, if a driver’s predicted target speed is above the speed limit, the model provides predictions of both intentional and unintentional speeding. Switching algorithms between these two types of speeding relies on a set of rules \( R_i = [R_1, R_2, ..., R_m] \) for \( m = 1, 2, ..., m \). For example, in the absence of traffic flow, the changes of posted speed limits serve as the switching points that trigger a driver’s act to step on the pedals. When the driver accelerates/decelerates to his/her target speed, the model will predict the occurrence of an intentional speeding in the near future. In a congested driving situation (there is interaction between vehicles), the headway spacing between the driver and the leader is another switching point \([44]\). When a person drives within a safe following distance (e.g., 50 m), the model switches to the intentional speeding mode. This is because other surrounding vehicles’ behaviors may affect the driver’s speed choice. For example, if a lead vehicle accelerates and exceeds the speed limit, the driver may follow it and eventually speed. In this case, speeding is intentional because the driver tends to follow the leader’s speed.

D. Warning Messages with Human Machine Interface (HMI)

The warning message is scheduled and presented via an in-vehicle HMI. ISPS always provides visual information about the current posted speed limit in force. When a driver exceeds the speed limit, the ISPS system warns the driver visually and verbally: the visual speed limit indicator is shown on an in-vehicle device (i.e., GPS) and increase in size and starts flashing. The verbal message, by default, is played with a female voice: “Be careful. Speed limit is XXX mph”. This message is repeated every 10 s until the speed is reduced to below the speed limit.

In addition, previous studies of ISA show that the acceptance of the ISA significantly increases if the threshold for warnings increases and the frequency of warnings decreases, but the effects on speeding behavior become smaller \([6]\). This indicates that finding a balance between the acceptance and effectiveness of system is quite crucial. In this study, ISPS allowed the driver to customize the threshold for warnings and warning messages (e.g., tones or voice recordings from his/her family members) to increase system acceptance \([45]\). Also, ISPS uses degraded warning messages based on a driver’s magnitude of speeding.

III. AN EXPERIMENTAL STUDY TO VALIDATE THE ISPS’S PREDICTIONS

This section describes our experimental study. A laboratory session involving a driving simulator was conducted to validate the ISPS’s predictions on speeding behavior.

A. Participants

Twenty-four participants (12 males, 12 females) ranging from 26 to 50 (M=33.2, SD=7.01) years of age took part in this
study. Their average driving experience was 6.17 years and average annual mileage was 9,330 miles. All of them have valid driver licenses and have driven for at least two years.

B. Self-report Measures

All participants were asked to complete three questionnaires after engaging in the driving task. The first questionnaire was designed to capture the participants’ demographic situation (such as age, gender, etc) and driving history (such as estimated cumulative driving mileage, the year a driver license was first issued, etc). Then, they were asked to construct a subjective value metric. Finally, a short form of the Revised Eysenck Personality Questionnaire [EPQR-S; 37] was administrated to divide all drivers into three categories: normal drivers (those characterized as E+ and N- or E- and N+, n=11), impulsive drivers (those characterized as E+ and N+, n=7), and non-impulsive drivers (those characterized as E- and N-, n=6).

C. Apparatus

A STISIM® driving simulator (STISIMDRIVE M100K) was used in the study. It includes a steering wheel with force feedback, a throttle pedal, and a brake pedal. The STISIM simulator was installed on a Dell Workstation (Precision 490, Dual Core Intel Xeon Processor 5130 2GHz) with a 256MB PCIe=16 nVidia graphic card, Sound Blaster® X-FiTM system, and Dell A225 Stereo System.

D. Driving Scenario

Test Blocks simulated a daily commuting environment consisting of 55% urban driving and 45% highway driving (two-lane in each direction). Each block was divided into three sections: the first 30% was urban driving, the middle 45% was highway driving, and the last 25% was urban driving again. During the two driving sections, a signalized intersection was repeatedly displayed (every 5,000 feet) 1,000 feet in front of the driver. Two speed limit signs (25 and 65 mph) were displayed 200 feet in front of the driver by turns. Accordingly, the acceleration process (25-65 mph) and the deceleration process (65-25 mph) repeated four times, respectively in two urban driving sections. In contrast, there was no signalized intersection in the section of highway driving and the speed limit was kept constant at 55 mph. Drivers were instructed to adjust their speed based on the speed limit as if they were driving a real vehicle on the road.

E. Procedures

Upon arrival, participants were first asked to sign a consent document. After completing a set of questionnaires, participants went through a Practice session that allowed them to get familiar with the driving simulator controls. The Practice session lasted for a half hour and contained all the features that appeared in the Test Blocks. Before the formal experiment, participants were allowed to adjust the seat so that they felt comfortable. In the Test Block session, each trial (12-mile) lasted for 15-20 min. The whole experiment lasted for 1.5-2 hours and the drivers’ participation time was compensated. Participants were asked to operate the driving simulator and follow the traffic laws as if they were driving a real vehicle on the road.

F. Measurement

The following behavioral measures were automatically collected from the driving simulator: time elapsed (unit in second), speed (m/s), acceleration (m/s²), gas pedal angle (degree), and brake pedal angle (degree). A speeding violation, in this experiment, was defined as 1 mph or more over the posted speed limit [46].

IV. VALIDATION OF THE ISPS’S PREDICTIONS WITH EXPERIMENTAL DATA

Both the beginning and ending urban sections focused on how a driver accelerated or decelerated to his/her target speed when the speed limit changed. Because this acceleration or deceleration process involves the cognitive behavior of decision making (i.e., setting a target speed based on the posted speed limit), speeding in these two blocks was intentional. On the other hand, the highway section focused on unintentional speeding while a driver maintaining a constant speed.

The prediction of intentional speeding consisted of the time at which a driver exceeded the speed limit and the magnitude of speeding. The modeled time at which a driver sped was compared with his/her experimental result in terms of the percentage of relative error (estimation error, [47-49])². Nonparametric (Mann-Whitney Test) tests were also performed to examine the differences (Δt) between model predictions and experimental data at an alpha level of 0.05. The modeled magnitude of speeding for individual driver was compared with his/her experimental result in terms of the root-mean-square (RMS) and estimation error (EE). Secondly, the signal detection theory (SDT) was used to evaluate the performance of the model’s predictions of unintentional speeding during the highway driving.

A. Prediction of Intentional Speeding

According to the results of RDFT, the modeled speed for those drivers who tended to exceed the speed limit was compared with their averaged experimental speeds (see Fig. 2a). A dashed line indicated the speeding criteria (the posted speed limit plus 1 mph). A dashed dot line indicated each driver’s target speed. Red reference lines represented the time at which drivers exceeded speeding criteria or their target speeds in the experiment. Blue ones represented the predicted occurrence of an intentional speeding during urban driving. Green ones indicated the time at which the model detected an unintentional speeding. After accelerating or decelerating to their target speeds, drivers tended to maintain constant speeds and these maintenance processes were omitted.

During the acceleration process, the time difference between model predictions and experimental data was, on average, 2 s

² The estimation error is derived based on |Y - X|/X * 100%. Y: modeled result; X: experimental result.
Nonparametric tests showed that there was no statistically significant time difference between modeled predictions and experimental results (U=25, Z=-0.74, p=0.51). The percentage of relative error was 12.6%. During the deceleration process, the time difference between model predictions and experimental data was 1.4 s (SD=1.01 s). The Mann-Whitney Test indicated that there was no statistically significant time difference between modeled and experimental results (U=21.5, Z=-1.11, p=0.29) (estimation error=13.7%). On the other hand, the averaged RMS of the modeled speeding magnitude was 0.85 m/s (SD=0.32 m/s) when drivers accelerated from 25 mph to 65 mph (estimation error=1.6%). While during the deceleration process (65-25 mph), the averaged RMS of the modeled speeding magnitude was 0.79 m/s (SD=0.29 m/s) (estimation error=4%).

**B. Prediction of Unintentional Speeding**

Signal detection theory (SDT) is applicable in any situation in which there are two discrete states of the world (signal and noise) that cannot easily be discriminated. In SDT, there are four classes of joint events, labeled hits, misses, false alarms, and correct rejections (see Fig. 3).

![Fig. 2a. Comparisons of predicted speeding with experimental data for Diver 2 (based on the median value of d’ in the group). This group of drivers (D2, D3, D5, D16, D17, D20, D21, and D22) tended to exceed the posted speed limit in both urban and highway sections. Only one acceleration process and one deceleration process during urban driving were provided as examples.](image)

![Fig. 2b. Comparisons of predicted speeding with experimental data for Driver 10 (based on the median value of d’ in the group). This group of drivers (D1, D4, D7, D8, D9, D10, D12, D14, D18, D19, D23, and D24) tended to follow the speed limit but occasionally failed. Only one acceleration process and one deceleration process during urban driving were provided as examples.](image)

These values are typically expressed as probabilities, by dividing the number of instances in a cell by the total number of instances in a column. In this study, HIT (hit rate) is the ratio of the number of instances the model predicts there is a speeding to the number of instances there is a true speeding in the experiment. FA (false alarm rate) is the ratio of the number of instances the model predicts there is a speeding to the number of instances there is no speeding in the experiment.

The model performance was evaluated with three different measures: 1) Testing accuracy which is the ratio of the number of instances correctly identified by the model to the total number of instances in the experiment; 2) Model sensitivity (d’); 3) Response bias (β), which were calculated according to

\[
d' = 
\Phi^{-1}(\text{HIT}) - \Phi^{-1}(\text{FA})
\]

\[
\beta = e^{\Phi^{-1}(\text{HIT}) - \Phi^{-1}(\text{FA})}
\]

Where \(\Phi^{-1}\) presents the function of calculating z-score and d’ represents the ability of the model to detect the occurrence of
speeding. The larger the value of d’ is, the more sensitive the model is. Note that β signifies the strategy used by the model. When β equals 1, models favor neither “prediction of a speeding” nor “prediction of no speeding”, and false alarms and misses tend to occur at similar rates. When β < 1, the models are classified as liberal and are more likely to overestimate the occurrence of speeding with higher false alarm rates than miss rates. When β > 1, the models are classified as conservative and are more likely to underestimate the occurrence of speeding with more misses than false alarms.

The summarizing parameters for model inputs are window size, overlap between windows and magnitude. Window size denotes the period over which a driver was examined if he/she is going to speed in near future. The comparisons of window size serve to identify the appropriate length of data that can be summarized to reduce the noise of the input data without losing useful information. The following four window sizes are chosen: 1, 3, 5, and 7 s. Overlap is the percentage of data that are shared between windows. It reflects the redundancy between instances and influences the detection frequency of models. The following three overlaps are chosen: 10%, 50%, and 90%. Magnitude refers to the deviation of the gas pedal input (dictated as \( \theta_p \) in Equation 8) within a window size. If the deviation of gas pedal input exceeds such a predefined magnitude (\( d\theta > \theta_p \)), the model predicts the time at which a driver will exceed the speed limit. The following six magnitudes are chosen: 0.5, 1, 1.5, 2, 2.5, and 3 degree. In short, window size, overlap, and magnitude interact to affect the computational load on the detection system (4 × 3 × 6 = 72 combinations in this study).

According to the gas pedal information, the model can predict the occurrence of unintentional speeding. 16 out of 24 drivers tended to follow the speed limit (the posted speed limit plus 1 mph) and 12 of them exhibited unintentional speeding behaviors (i.e., speed repeatedly moved up and down around the speed limit). For these 12 drivers, the model predicted the time at which they exceed the speed limit (see Fig. 2b). The optimal SDT results were provided in Table II: the average d’ was 1.81 (range = 1.21 - 2.46) and the average testing accuracy was 86% (range = 78.6% - 93.1%). The model predicted the speeding, on average, 4.43 s prior to its occurrence in the experiment. Also, the model took a conservative strategy for 9 drivers (β > 1) while a risky strategy for the other 3 drivers (β < 1).

On the other hand, although 8 out of 12 drivers always exceeded the speed limit by a larger magnitude, the model can still predict the time at which they exceeded their target speeds (see Fig. 2a). The optimal SDT results were also provided in Table II: the average d’ was 2.42 (range = 1.31 - 5.15) and the average testing accuracy was 87.4% (range = 78.1% - 93%). The model predicted the speeding, on average, 4.82 s prior to its occurrence in the experiment. The model took a conservative strategy for 7 drivers while a risky strategy for the other 1 driver.
TABLE II. RMS AND ESTIMATION ERROR RESULTS FOR MODELED MAGNITUDE OF SPEEDING

<table>
<thead>
<tr>
<th>Group</th>
<th>Driver</th>
<th>Predicted DMR (mph)</th>
<th>Intentional speeding</th>
<th>Unintentional speeding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Δt (s)</td>
<td>EE</td>
<td>RMS (m/s)</td>
</tr>
<tr>
<td>Drivers who exceeded the speed limit (n=8)</td>
<td>D2</td>
<td>10</td>
<td>0.5</td>
<td>4.8%</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>5</td>
<td>2</td>
<td>8.9%</td>
</tr>
<tr>
<td></td>
<td>D5</td>
<td>5</td>
<td>1</td>
<td>5.7%</td>
</tr>
<tr>
<td></td>
<td>D16</td>
<td>3</td>
<td>2.5</td>
<td>16.9%</td>
</tr>
<tr>
<td></td>
<td>D17</td>
<td>3</td>
<td>1.2</td>
<td>10.3%</td>
</tr>
<tr>
<td></td>
<td>D20</td>
<td>5</td>
<td>3.6</td>
<td>20.7%</td>
</tr>
<tr>
<td></td>
<td>D21</td>
<td>2</td>
<td>2</td>
<td>14.3%</td>
</tr>
<tr>
<td></td>
<td>D22</td>
<td>5</td>
<td>3.2</td>
<td>19.5%</td>
</tr>
<tr>
<td>Average within this group</td>
<td>2</td>
<td>12.6%</td>
<td>0.85</td>
<td>1.6%</td>
</tr>
<tr>
<td>Drivers who tended to follow the speed limit but failed occasionally (n=12)</td>
<td>D1</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D4</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D7</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D8</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D9</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D10</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D12</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D14</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D18</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D19</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D23</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D24</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Average within this group</td>
<td>1.81</td>
<td>86%</td>
<td>1.49</td>
<td>4.43</td>
</tr>
<tr>
<td>Drivers who followed the speed limit (n=4)</td>
<td>D6</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D11</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D13</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>D15</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

V. DISCUSSION

This work has extended previous mathematical model of driver speed control to provide quantitative predictions of intentional and unintentional speeding for individual drivers. These predictions consist of the time at which the driver exceeds the speed limit and the magnitude of speeding. Based on this new extended work, we have developed an intelligent speeding prediction system (ISPS) to prevent the occurrence of speeding.

The previous model of driver speed control integrates speed perception, decision making, motor control, and vehicle mechanics in a cohesive manner to model the average driver’s behavior of speed control. In this study, we have modeled two individual factors (decision making references and impulsiveness) and their effects on a driver’s decision making and motor control. Given this new model component (i.e., individual differences), ISPS can predict speeding for not only the average driver, but also for individual drivers. Additionally, the previous model of driver speed control requires the cognitive behavior of decision making on speed choice (i.e., setting a target speed). Therefore, it cannot predict an unintentional speeding which may result from a lack of awareness of the current speed limit and/or travelling speed, or an inaccurate foot movement required for pedal operation [5]. To solve this problem, this work has used the deviation of the pedal input to predict unintentional speeding. If such deviation exceeds a predefined threshold, ISPS calculates the time at which a driver will speed and schedules a speeding warning message.

An experimental study involving a driving simulator has been conducted to evaluate ISPS. We found that there was no difference between modeled predictions (intentional speeding) and experimental data, in terms of the time and magnitude of speeding. The ISPS was sensitive (the average d’ was 2.1) and accurate (the average testing accuracy was over 86%) to predict the majority of unintentional speeding with a relatively small portion of unnecessary speeding warnings. The ISPS was able to predict unintentional speeding more than 4 seconds prior to its occurrence. The 4 seconds is enough to play a warning and respond for a normal driver [50]. Moreover, choosing summarizing parameters (window size, overlap and magnitude) for input data is important for a real-time speeding detection. Speed control is a normal vehicle control involved in daily driving. With successive practice, a driver should form his/her own driving pattern. This allows ISPS to choose the optimal
parameters for each driver with the best model performance. Also, this will significantly reduce the model’s computational load, which allows the implementation of ISPS in real-time speeding detection and prevention.

Compared to existing theoretical approaches used to predict speeding (e.g., TPB, TCI), ISPS shows its unique advantages. First, ISPS is developed based on a set of mathematical equations. Given these math equations and parameters, anyone can implement the model to get the same predictions of speeding. Second, ISPS (mathematical model) can be easily embedded in an in-vehicle computer system and provide real-time speeding predictions. Third, this work is the first mathematical model with analytic solutions that predict speeding (both intentional and unintentional) for not only the average driver, but also for individual drivers. Brandenburg and Drewitz [20] propose a mathematical equation to describe the amount of vehicle acceleration as a function of the difference between capability and task demand [20]. However, these authors do not validate the model’s predictions of speeding (such as the time at which a driver exceeds the speed limit or the magnitude of speeding) with experimental or empirical data in their study.

Application of ISPS has practical value for reducing driving speed and preventing the occurrence of real-time speeding and speeding-related traffic accidents. Most existing studies on the effects of ISA on speeding behaviors show that ISA reduces the mean speed and speeding violations [3-6]. Unlike ISA, which is a post-feedback warning system, ISPS provides the driver with a warning message before the act of speeding occurs. If a driver pays attention to the warning message, he/she should always drive below the speed limit. Therefore, ISPS may achieve higher reduction in driving speed than ISA. More importantly, a driver may benefit from the ISPS by avoiding a speeding-related crash. The NHTSA identifies crash imminent test scenarios based on common pre-crash scenarios which describe vehicle movements and critical events immediately prior to the crash [51]. Some of these scenarios result from speeding. For example, a host vehicle is following a lead vehicle at the same speed, both driving too fast for the upcoming curve. The lead vehicle suddenly loses control and decelerates. The host vehicle does not have enough spacing to stop and may collide with the lead vehicle. In this case and similar cases that involve an imminent crash due to speeding, ISA may not help a driver to avoid the collision: the host driver may not have enough time or distance to stop when he/she already exceeds the speed limit. In contrast, ISPS predicts the occurrence of speeding and provides the driver with a warning message prior to it. This allows a driver to reduce speed in advance (i.e., lower speed means more time and distance to stop in emergency) and avoid such collisions.

In addition, if ISPS predicts that a driver is going to speed, it can send warning messages to other surrounding drivers via broadcasting or advanced wireless technologies [52]. This assumes that all vehicles have equipped with the ISPS. In the last example, if ISPS predicts that the lead vehicle will speed, it can send a warning message (e.g., “the lead vehicle is going to speed”) to the host driver so that he/she can pay more attention on that vehicle, and respond quickly when it suddenly decelerates.

Even though this work validates the effectiveness of the ISPS in predicting and preventing speeding, it remains unclear how acceptable such a warning system will be. A driver may turn off the system frequently if receiving an annoying number of anticipatory warnings. Future experimental or empirical study is needed to assess the acceptance of the ISPS over extended periods of time before its widespread implementation [53]. In addition, it might be beneficial to compare the effectiveness of the ISPS (a proactive speeding warning system) with ISA (a post-feedback speeding warning system) in reducing speed and speeding behaviors. Finally, factors such as driving experience may affect a driver’s behavior of speed control. Due to page limit, however, it is extremely hard to model all of these aspects in one study and we are extending the model to consider these important factors in our future work.

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