

For a positive integer *n*, the number of positive integers less than or equal to *n* and relatively prime to *n*.

Primitive *n*-th roots of unity

Let *n* be a positive integer and let .

The quantities , where  and *gcd(n,k)=1*.

*n*-th cyclotomic polynomial over *Q*

For positive integer *n*, let  be the primitive roots of unity.

The polynomial .

Theorem 33.1

For every positive integer *n*, , where the product runs over all positive divisors *d* of *n*.

Theorem 33.2

For every positive integer *n*,  has integer coefficients.

Theorem 33.3 (Gauss)

The cyclotomic polynomials  are irreducible over *Z*.

Theorem 33.4

Let  be a primitive *n*-th root of unity. Then .

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Let *n* be a positive integer and let . Then .

Theorem 33.5 (Gauss)

It is possible to construct the regular *n*-gon with a straightedge and compass if and only if *n* has the form , where  and the  are distinct primes of the form . [Note: if  is prime, with , it can be shown that *m* is a power of 2.]