Automorphism of *E*

*E* is an extension of field *F*.

A ring isomorphism from *E* onto *E*.

Galois Group of *E* Over *F*

The set of all automorphisms of *E* that take every element of *F* to itself.

Denoted .

Fixed field of *H*

*H* is a subgroup of .

The set .

Fundamental Theorem of Galois Theory

(Part 1 of 2)

Let *F* be a field of characteristic 0 or a finite field. If *E* is the splitting field over *F* for some polynomial in , then the mapping from the set of subfields of *E* containing *F* to the set of subgroups of  given by  is a one-to-one correspondence. Furthermore, for any subfield *K* of *E* containing *F*,

1.  and  . [The index of  in  equals the degree of *K* over *F*.]

Fundamental Theorem of Galois Theory

(Part 2 of 2)

1. If *K* is the splitting field of some polynomial in , then  is a normal subgroup of  and  is isomorphic to .
2. . [The fixed field of  is *K*.]
3. If *H* is a subgroup of , then . [The automorphism group of *E* fixing  is *H*.]

Solvable by Radicals Over *F*

Let *F* be a field, and .

 splits in some extension  of *F* and there exist positive integers  such that  and  for .

Solvable Group

A group *G* has a series of subgroups , where for each ,  is normal in  and  is Abelian.

Splitting Field of 

Let *F* be a field of characteristic 0 and let . If *E* is the splitting field of  over *F*, then the Galois group  is solvable.

Factor Group of a Solvable Group Is Solvable

A factor group of a solvable group is solvable.

*N* and *G/N* Solvable Implies *G* Is Solvable

Let *N* be a normal subgroup of a group *G*. If both *N* and *G/N* are solvable, then *G* is solvable.

Solvable by Radicals Implies Solvable Group

Let *F* be a field of characteristic 0 and let . Suppose that  splits in , where

 and  for . Let *E* be the splitting field for  over *F* in . Then the Galois group  is solvable.